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Attractor-like dynamics in belief updating in schizophrenia

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2 updating in schizophrenia

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34 **Abstract**

35

36 Subjects with a diagnosis of schizophrenia (Scz) overweight unexpected
37 evidence in probabilistic inference: such evidence becomes ‘aberrantly salient’. A
38 neurobiological explanation for this effect is that diminished synaptic gain (e.g.
39 hypofunction of cortical *N*-methyl-D-aspartate receptors) in Scz destabilizes
40 quasi-stable neuronal network states (or ‘attractors’). This attractor instability
41 account predicts that i) Scz would overweight unexpected evidence but
42 underweight consistent evidence, ii) belief updating would be more vulnerable
43 to stochastic fluctuations in neural activity, and iii) these effects would correlate.

44

45 Hierarchical Bayesian belief updating models were tested in two independent
46 datasets (n=80 and n=167, male and female) comprising human subjects with
47 schizophrenia, and both clinical and non-clinical controls (some tested when
48 unwell and on recovery) performing the ‘probability estimates’ version of the
49 beads task (a probabilistic inference task). Models with a standard learning rate,
50 or including a parameter increasing updating to ‘disconfirmatory evidence’, or a
51 parameter encoding belief instability were formally compared.

52

53 The ‘belief instability’ model (based on the principles of attractor dynamics) had
54 most evidence in all groups in both datasets. Two of four parameters differed
55 between Scz and non-clinical controls in each dataset: belief instability and

56 response stochasticity. These parameters correlated in both datasets.
57 Furthermore, the clinical controls showed similar parameter distributions to Scz
58 when unwell, but were no different to controls once recovered.

59

60 These findings are consistent with the hypothesis that attractor network
61 instability contributes to belief updating abnormalities in Scz, and suggest that
62 similar changes may exist during acute illness in other psychiatric conditions.

63

64 **Significance Statement**

65

66

67 Subjects with a diagnosis of schizophrenia (Scz) make large adjustments to their
68 beliefs following unexpected evidence, but also smaller adjustments than
69 controls following consistent evidence. This has previously been construed as a
70 bias towards ‘disconfirmatory’ information, but a more mechanistic explanation
71 may be that in Scz, neural firing patterns (‘attractor states’) are less stable and
72 hence easily altered in response to both new evidence and stochastic neural
73 firing. We model belief updating in Scz and controls in two independent datasets
74 using a hierarchical Bayesian model, and show that all subjects are best fit by a
75 model containing a belief instability parameter. Both this and a response
76 stochasticity parameter are consistently altered in Scz, as the unstable attractor
77 hypothesis predicts.

78

79 **Introduction**

80

81

82 Subjects with a diagnosis of schizophrenia (Scz) tend to use less evidence to
83 make decisions in probabilistic tasks than healthy controls (Garety et al., 1991;
84 Dudley et al., 2016). The paradigm most commonly used to demonstrate this
85 effect is the 'beads' or 'urn' task, in which subjects are shown two urns, each
86 containing opposite ratios of coloured beads (e.g. 85% blue and 15% red and
87 vice versa), which are then hidden. A sequence of beads is then drawn (with
88 replacement) from one urn, and the subject either has to stop the sequence when
89 they are sure which urn it is coming from (the 'draws to decision' task) or the
90 subject must rate the probability of the sequence coming from either urn after
91 seeing each bead, without having to make any decision (the 'probability
92 estimates' task). Bayesian analysis of these tasks has indicated that Scz are more
93 stochastic in their responding (Moutoussis et al., 2011) and that they overweight
94 recent evidence and thus update their beliefs (in the probabilistic sense) more
95 rapidly (Jardri et al., 2017).

96 Several belief-updating abnormalities have been found in Scz using the
97 'probability estimates' task. The most consistent finding is that Scz (or just Scz
98 with delusions (Moritz and Woodward, 2005)) change their beliefs *more* than
99 non-psychiatric controls in response to changes in evidence (Langdon et al.,
100 2010) – particularly 'disconfirmatory' evidence, i.e. evidence contradicting a
101 current belief (Garety et al., 1991; Fear and Healy, 1997; Young and Bentall,

102 1997; Peters and Garety, 2006). Another is that probability ratings at the start of
103 the sequence are higher in currently psychotic (but not in recovered) Scz than in
104 both clinical and healthy controls (Peters and Garety, 2006), similar to the
105 ‘jumping to conclusions’ bias in the ‘draws to decision’ version of the task. Others
106 have also found that Scz update *less* than controls to more *consistent* evidence, in
107 this (Horga, in preparation) and other paradigms (Averbeck et al., 2010).

108 These findings can potentially be understood in the light of the ‘unstable
109 attractor network’ hypothesis of Scz. An attractor network is a neural network
110 that can occupy numerous stable states that are learned from experience, via
111 adjustments to synaptic weights. It can revisit these states if presented with
112 inputs that resemble previous patterns of synaptic weights, or through
113 spontaneous fluctuations in neural activity: either way, the activity of all nodes is
114 ‘attracted’ to a quasi-stable state because the network energy is lower at these
115 states, and network firing patterns evolve to minimise energy. Attractor
116 networks were originally developed to model the storage and reactivation of
117 memories (Hopfield, 1982), but related network models also offer mechanistic
118 explanations for working memory storage (e.g. Brunel and Wang, 2001),
119 decision-making (Wang, 2013) and interval timing (Standage et al., 2013), as
120 well as Bayesian belief updating (Gepperth and Lefort, 2016).

121 In Scz, attractor states in prefrontal cortex are thought to be less stable, so
122 it is easier for the network to switch between them, but harder to become more
123 confident about (i.e. increase the stability of) any particular one (Rolls et al.,
124 2008). This loss of stable neuronal states – recently demonstrated in two animal
125 models of Scz (Hamm et al., 2017) – is thought to be due to hypofunction of *N*-
126 methyl-D-aspartate receptors (NMDARs) or cortical dopamine 1 receptors in Scz

127 (Figure 1). Interestingly, healthy volunteers given ketamine (an NMDAR
128 antagonist) show a decrement in updating to consistent stimulus associations
129 and an increase in decision stochasticity in this context (Vinckier et al., 2016).
130 Attractor network perturbations have been linked to working memory problems
131 in Scz using a bistable (i.e. a stable ‘up’ state corresponding to persistent
132 neuronal activity, and a ‘down’ state corresponding to background activity)
133 model (Murray et al., 2014), but not as yet to a computational understanding of
134 belief updating.

135 We analysed belief updating in Scz using the Hierarchical Gaussian Filter
136 (HGF; Mathys et al., 2011), a variational Bayesian model with individual priors,
137 in two independent ‘probability estimates’ beads task datasets. We asked: given
138 the larger belief updates in Scz compared with controls, can these be explained
139 by group differences in i) general learning rate and/or ii) response stochasticity,
140 or by adding parameters encoding iii) the variance (i.e. uncertainty) of beliefs at
141 the start of the sequence, iv) a propensity to overweight disconfirmatory
142 evidence specifically, or v) patterns of belief updating typical of unstable
143 attractor states in a Hopfield-type network, i.e. greater instability and
144 stochasticity, which correlate with each other? (Note that the HGF does not
145 contain attractor states: the model in (v) is designed to simulate the effects on
146 inference that unstable neuronal attractors may have.) Furthermore, are these
147 findings consistent within Scz tested at different illness phases, and are they
148 unique to Scz or also present in other non-psychotic mood disorders?

149 **Methods and Materials**

150 **Subject characteristics**

151

152 Dataset 1 comprised 23 patients with delusions (18 Scz), 22 patients with non-
153 psychotic mood disorders, and 35 non-clinical controls (overall, 50 male and 30
154 female – see Table 1 for details of the groups); the first two groups were selected
155 from inpatient wards at the Maudsley and the Bethlem Royal Hospitals. All
156 groups were tested twice (with loss of n=25 from the groups – see Table 1); the
157 clinical groups were tested once when they were unwell ('baseline'), and again
158 once they had recovered ('follow-up'). The mean time between testing sessions
159 was 17.4 (range 6 to 41) weeks in the deluded group, 33.4 (range 4 to 68) weeks
160 in the clinical control group, and 35.6 (range 27 to 46) weeks in the non-clinical
161 control group. The deluded group's shorter inter-test interval was due to their
162 shorter admission period and to the prioritization of their follow-up over the
163 non-clinical control group. Dataset 1 is described in detail elsewhere (Peters and
164 Garety, 2006).

165 Dataset 2 comprised 56 subjects with a diagnosis of schizophrenia (Scz)
166 and 111 controls (overall, 83 male and 84 female – see Table 1). All subjects
167 provided informed, written consent, and ethical permission for the study was
168 obtained from the local NHS Research Ethics Committee (Reference
169 14/LO/0532). Given the National Adult Reading Test (Nelson, 1982) was used to
170 estimate IQ in these participants, a recruitment condition was that English was
171 their first language.

172 Measures of cognitive function and delusion-proneness (or schizotypy)
173 were collected in all subjects; clinical symptom ratings were collected in clinical
174 subjects only (see Table 1 for details).

175

176 **Experimental design**

177

178 Subjects in dataset 1 performed the ‘probability estimates’ beads task as used
179 previously (Garety et al., 1991), with two urns with ratios of 85:15 and 15:85
180 blue and red beads respectively, and viewing a single sequence of ten beads
181 (Figure 2); after each bead they had to mark an analogue scale (from 1 to 100)
182 denoting the probability the urn was 85% red.

183 Subjects in dataset 2 performed the ‘probability estimates’ beads task,
184 with two urns with ratios of 80:20 and 20:80 red and blue beads respectively.
185 They each viewed four separate sequences (two identical pairs of sequences with
186 the colours swapped within each pair) of ten beads (Figure 2); after each bead
187 they had to mark a Likert scale (from 1 to 7) denoting the probability the urn
188 was the 80% blue one. Two sequences contained an apparent change of jar. The
189 order of the four sequences was randomised.

190 We used some of the behavioural measures employed in the original
191 analysis of dataset 1 (Peters and Garety, 2006) to analyse dataset 2. These were
192 ‘disconfirmatory updating’, the mean change in belief on seeing a bead of a
193 different colour to the ≥ 2 beads preceding it and ‘final certainty’ (the response to
194 the last bead). We altered their ‘initial certainty’ measure from the mean
195 response to the first three beads to the response to the first bead, which comes

196 closer to capturing the classic ‘jumping to conclusions’ bias (in which around
197 50% of Scz decide on the jar colour after seeing only one bead; (Garety et al.,
198 1991), although the results of both measures are presented below.

199

200 **Computational modelling**

201

202 The optimal way to use sensory information to update one’s beliefs under
203 conditions of uncertainty is to use Bayesian inference. Neural systems are likely
204 to approximate Bayesian inference using schemes of simple update equations
205 (Rao and Ballard, 1999; Friston, 2005); one such model is the Hierarchical
206 Gaussian Filter (HGF). The HGF is a hierarchical Bayesian inference scheme that
207 gives a principled account of how beliefs are updated on acquiring new data,
208 using variational Bayes and individual priors. Variational Bayesian schemes (e.g.
209 (Beal, 2003) use analytic equations to derive an exact solution to an
210 approximation of the posterior distribution over the latent variables and
211 parameters (as opposed to sampling methods which approximate a solution to
212 the exact posterior). The HGF has been used as a generic state model for learning
213 under uncertainty and has repeatedly been shown to outperform similar
214 approaches, such as reinforcement learning models with fixed (e.g. Rescorla-
215 Wagner) or dynamic (e.g. (Sutton, 1992) learning rates (Iglesias et al., 2013;
216 Diaconescu et al., 2014; Hauser et al., 2014; Vossel et al., 2014). One advantage of
217 the HGF is that it contains subject-specific parameters (and prior beliefs) that
218 can account for between-subject differences in learning whilst preserving the
219 (Bayes) optimality of any individual’s learning (relative to his/her model

220 parameters and prior beliefs). These parameters may be encoded by tonic levels
221 of neuromodulators such as dopamine (Marshall et al., 2016), or by the intrinsic
222 properties of neuronal networks (e.g. the ratio of excitatory to inhibitory neural
223 activity can affect both the speed of evidence accumulation (Lam et al., 2017) –
224 analogous to the evolution rate in the HGF – and also response stochasticity
225 (Murray et al., 2014)). Differences in model parameters between Scz and
226 controls may therefore explain, in computational terms, how pathophysiology
227 leads to abnormal inference (Adams et al., 2015).

228 In general, when modelling behaviour under Bayesian assumptions, it is
229 necessary to distinguish between the model of the world used by the subject (the
230 perceptual model) and a model of how a subject's beliefs translated into
231 observed behaviour (the observation or response model). Most of the
232 parameters pertain to the perceptual model (here, all parameters except
233 response stochasticity v – see Table 2) and reflect (inferred) neuronal
234 processing. In contrast, the parameters of the response model link subjective
235 states to behavioural outcomes, and thus may reflect stochasticity in neuronal
236 processing, measurement noise (in some paradigms), or non-random effects that
237 have not been captured by the perceptual model. This and related learning
238 models are freely available from
239 <http://www.translationalneuromodeling.org/tapas/> (version 5.1.0): this
240 analysis used the perceptual models 'hgf_binary' or 'hgf_ar1_binary' and the
241 response model 'beta_obs'.

242 At the bottom of the model (Figure 3 shows some simulated responses) is
243 the bead drawn $u^{(k)}$ on trial k and the probability $x_1^{(k)}$ that draws are coming
244 from the blue jar. At the level above this is x_2 , the tendency towards the blue jar

245 (a transform of the probability, bounded by $\pm\infty$); by definition, $x_1 = s(x_2)$, where
 246 $s(\bullet)$ is the logistic sigmoid function. As x_2 approaches infinity, the probability of
 247 the blue jar approaches 1; as it approaches minus infinity, the probability of the
 248 blue jar approaches 0. For $x_2 = 0$, both jars are equally probable. This quantity is
 249 hidden from the subject and must be inferred: the subject's posterior estimate of
 250 x_2 is μ_2 , and the subject's posterior estimate of the probability of the jar being
 251 blue on trial k is $s(\mu_2^{(k)})$ – equivalent to the prediction (denoted by $\hat{\mu}_1$) on the next
 252 trial $\hat{\mu}_1^{(k+1)}$.

253 Before seeing any new input on trial k the model's expected jar
 254 probability $\hat{\mu}_1^{(k)}$ and precisions (inverse variances) $\hat{\pi}_1^{(k)}, \hat{\pi}_2^{(k)}$ of the expectations
 255 at each level are given by:

$$256 \quad \hat{\mu}_1^{(k)} \equiv s(\kappa_1 \mu_2^{(k-1)})$$

$$257 \quad \hat{\pi}_1^{(k)} \equiv \frac{1}{\hat{\mu}_1^{(k)}(1 - \hat{\mu}_1^{(k)})}$$

$$258 \quad \hat{\pi}_2^{(k)} \equiv \frac{1}{\sigma_2^{(k-1)} + \exp(\omega)}$$

259 Note that in Models 1-4, κ_1 is fixed to 1. A new input $u^{(k)} \equiv \mu_1^{(k)}$ generates
 260 a prediction error $\delta_1^{(k)}$ and the model updates and generates a new prediction as
 261 follows:

$$262 \quad \delta_1^{(k)} \equiv \mu_1^{(k)} - \hat{\mu}_1^{(k)}$$

$$263 \quad \pi_2^{(k)} = \hat{\pi}_2^{(k)} + \frac{\kappa_1^2}{\hat{\pi}_1^{(k)}}$$

$$264 \quad \mu_2^{(k)} = \mu_2^{(k-1)} + \frac{\kappa_1}{\pi_2^{(k)}} \delta_1^{(k)}$$

$$265 \quad \hat{\mu}_1^{(k+1)} \equiv s(\kappa_1 \mu_2^{(k)})$$

266 The subject's response $y^{(k)}$ (i.e. where on the continuous or Likert scale
267 they responded) is determined by $\hat{\mu}_1^{(k+1)}$ and the precision of the response
268 model's beta distribution ν .

269 We parameterize the beta distribution in terms of its mean μ and
270 precision ν . These sufficient statistics relate to the conventional
271 parameterization in terms of the sufficient statistics α and β by the following
272 bijection:

$$273 \quad \mu := \frac{\alpha}{\alpha + \beta}$$

$$274 \quad \nu := \alpha + \beta$$

275 Note that updates to μ_2 are driven by the product of the prediction error
276 from Bayesian updating explained above and a learning rate which, crucially, can
277 change over time: this is an important aspect of the HGF in contrast to learning
278 models such as Rescorla-Wagner that have a fixed learning rate. Parameters
279 which affect the degree to which μ_2 can change during the experiment include ω ,
280 φ , κ_1 and $\sigma_2^{(0)}$. The contributions of φ and κ_1 are illustrated in Figure 4 (left
281 panels).

282 The model usually has a third level, at which x_3 encodes the phasic
283 volatility of x_2 (this determines the probability of the jar changing at any point):
284 given the very short sequences employed in our datasets, from which volatility
285 cannot be reliably estimated, we omitted this level. In any case, volatility could
286 not account for the rapid changes in learning rate (from trial to trial, following
287 confirmatory vs disconfirmatory evidence) present in the Scz group in these
288 datasets.

289 In Models 1 and 2, changes in x_2 from trial to trial occur only according to
 290 the evolution rate ω , the variance of the random process at the second level.
 291 These models were equivalent to the subsequent models with either φ (Models 3
 292 and 4) fixed to 0 or κ_1 (Models 5 and 6) fixed to 1.

293 In Models 3 and 4, changes in x_2 from trial to trial occur according to an
 294 autoregressive (AR(1)) process that is controlled by three parameters: m , the
 295 level to which x_2 is attracted, φ , the rate of change of x_2 towards m , and ω , the
 296 variance of the random process:

$$297 \quad p(x_2^{(k+1)}) \sim \mathcal{N}(x_2^{(k)} + \varphi(m - x_2^{(k)}), \exp(\omega))$$

298 After inversion, the evolution of x_2 according to this equation is reflected in the
 299 prediction of μ_2 :

$$300 \quad \hat{\mu}_2^{(k+1)} = \mu_2^{(k)} + \varphi(m - \mu_2^{(k)})$$

301 In this study, given there was no bias towards one jar or the other, m was
 302 fixed to 0, so φ always acted to shift the model's beliefs back towards maximum
 303 uncertainty (i.e. disconfirm the current belief) about the jars. Figure 4 (upper left
 304 panel) illustrates the effect of φ on $s(\mu_2^{(k)})$ over time.

305 In Models 5 and 6, changes in μ_2 from trial to trial occur according to two
 306 parameters: ω , the variance of the random process, and κ_1 , a scaling factor that
 307 changes the size of updates when $\hat{\mu}_1 = 0.5$, or maximum uncertainty, relative to
 308 when $\hat{\mu}_1$ is closer to 0 or 1, i.e. when the subject is more confident about either
 309 jar. Figure 4 (lower left panel) illustrates the effect of κ_1 on $\hat{\mu}_1$ over time.

310 Formally, the scaling occurs as:

$$311 \quad \hat{\mu}_1^{(k+1)} \equiv s(\mu_2^{(k)} \kappa_1)$$

312 When $\kappa_I > 1$, updating towards 1 on observing a blue bead ($u = 1$) is
313 greatest (i.e. switching between jars becomes more likely) when $\hat{\mu}_1 < 0.3$; when
314 $\kappa_I < 1$, updating is comparatively far lower when $\hat{\mu}_1 < 0.3$. This is illustrated in
315 Figure 4 (middle panel): for high values of κ_I (brown line), belief updates that
316 cross the $\hat{\mu}_1 = 0.5$ line encounter little resistance (i.e. little evidence is required
317 to cause a large shift), while approaching the extremes of $\hat{\mu}_1 = 0$ and $\hat{\mu}_1 = 1$ in
318 response to confirmatory evidence is resisted (belief shifts are very small for $\hat{\mu}_1$
319 near 1). By contrast, for low values of κ_I (black line, Figure 4 middle panel), there
320 is relatively less resistance against approaching the extremes while it takes more
321 evidence for beliefs to cross the $\hat{\mu}_1 = 0.5$ line.

322 Figure 4 (right panel) illustrates the average absolute shifts in beliefs on
323 observing beads of either colour. This ‘vulnerability to updating’ is highly
324 reminiscent of the ‘energy state’ of a neural network model – i.e. in low energy
325 states, less updating occurs. The effect of increasing κ_I is to convert confident
326 beliefs about the jar (near 0 and 1) from low to high ‘energy states’, i.e. to make
327 them much more unstable. This recapitulates the attractor network properties
328 illustrated in Figure 1: an unstable network easily switches from one state to
329 another but has difficulty stabilising any one state, whereas a stable network
330 requires more energy (here, information) to overcome the boundary between
331 two states (here, beliefs). Models 5 and 6 therefore capture the effects of
332 attractor (in)stability on belief updating, or at least the kind of updating for
333 which (un)stable attractor states are a good analogy.

334 As group differences in initial updating had been observed in dataset 1,
335 we also estimated the standard deviation of μ_2 before the sequence begins, $\sigma_2^{(0)}$,
336 in Models 2, 4 and 6.

337 NB for intermediate values of κ_1 , Models 5 and 6 produce similar belief
338 updating trajectories to Models 3 and 4 (containing the disconfirmatory updating
339 parameter φ): both make greater updates following disconfirmatory evidence.
340 For more extreme values of κ_1 , however, Models 5 and 6 produce trajectories
341 that Models 3 and 4 cannot: φ cannot pull beliefs far towards certainty in the
342 opposite jar (c.f. brown line in Figure 4, lower left panel), and neither can it make
343 it *more* difficult to update to disconfirmatory evidence (c.f. black line in Figure 4,
344 lower left panel).

345 The parameters ω and $\nu \pm \sigma_2^{(0)} \pm \varphi$ or κ_1 were estimated individually
346 for each subject. If estimated, the prior probability distributions for their values
347 are given in Table 2. The means given here refer to the parameters' native space,
348 but the variances refer not to the parameters' native space, which in many cases
349 is bounded, but to the unbounded space they were transformed to for estimation
350 purposes. Otherwise they were fixed as $\varphi = 0$ (Models 1 and 2) and $\sigma_2^{(0)} =$
351 0.006 (Models 1, 3 and 5). The model's prior beliefs about the jars at the start of
352 the sequence were fixed at $\mu_2^{(0)} = 0$ (i.e. believing each to be equally likely). The
353 priors were sufficiently uninformative to be easily updated by the data: all prior
354 means are standard for the HGF except $\sigma_2^{(0)}$, which had to be increased from
355 0.006 to 0.8 to allow the data to change it. The latter change ensured that group
356 differences in initial belief updating alone would cause group differences in $\sigma_2^{(0)}$
357 rather than κ_1 .

358

359 **Model fitting and statistical analysis**

360

361 We tested models with different combinations of parameters ω , ν , φ or κ_1 and
362 $\sigma_2^{(0)}$ (see Table 2). In analysing dataset 2, we concatenated all four sequences for
363 each subject in order to estimate the model parameters as accurately as possible
364 (resetting the beliefs about the jars at the start of each sequence).

365 After fitting the six models to each subject's data, we performed Bayesian
366 model selection on all groups separately in both dataset 1 (at baseline and
367 follow-up) and dataset 2. This procedure weights models according to their
368 accuracy but penalises them for complexity (i.e. unnecessary extra parameters)
369 to prevent overfitting (Stephan et al., 2009; Rigoux et al., 2014). The winning
370 model in all eight groups was Model 6 (Figure 6), although around a third of
371 psychotic subjects and non-clinical controls in dataset 1 (at baseline) and in
372 dataset 2 were better fit by Model 4. It is unclear why this change occurs, but
373 given that Model 6 can produce very similar trajectories to Model 4 for
374 intermediate values of κ_1 (Figure 4), any increase in response stochasticity is
375 likely to diminish the strength of evidence for one model over a similar one.

376 In order to confirm we could reliably estimate the parameters of the
377 winning model, Model 6, we simulated 100 datasets using the modal values of
378 the parameters for both control and Scz groups (Figure 5, upper and lower rows
379 respectively; an example simulated dataset is shown in Figure 3). We then
380 estimated the parameters for the simulated data, and showed that in most cases,
381 the parameters are recovered reasonably accurately. The exception was $\sigma_2^{(0)}$ in
382 the Scz group simulation, which was distributed around the prior mean of 0.8
383 rather than the true value of 1.5. We retained a prior mean of 0.8 for $\sigma_2^{(0)}$
384 because using a higher prior mean led to overestimation of $\sigma_2^{(0)}$ in other
385 simulations (not shown).

386

387 **Results**

388

389 **Behavioural results: dataset 1**

390

391 Each group's mean responses are plotted in Figure 2A, and statistical tests
392 detailed in Table 1 ($p(adj)$ refers to the adjusted p value of Tukey's HSD *post hoc*
393 test). As described previously (Peters and Garety, 2006), at baseline there was a
394 significant difference in disconfirmatory updating between the groups ($F(2,77) =$
395 $6, p = 0.004$, ANOVA), and the psychotic group had greater disconfirmatory
396 updating than the non-clinical controls ($p(adj) = 0.003$) but not the clinical
397 controls ($p(adj) = 0.4$). There was no difference between the clinical and non-
398 clinical controls ($p(adj) = 0.13$). There were also significant differences in initial
399 certainty across the three groups ($F(2,77) = 8.7, p = 0.0004$, ANOVA); the
400 psychotic group's initial certainty was higher than the non-clinical controls'
401 ($p(adj) = 0.0003$) but not the clinical controls' ($p(adj) = 0.25$). There wasn't a
402 significant difference between the clinical and non-clinical control groups ($p(adj)$
403 $= 0.06$). There were no group differences in final certainty ($F(2,77) = 0.7, p = 0.5$,
404 ANOVA).

405 At follow-up, the difference in disconfirmatory updating between the
406 groups was no longer significant ($F(2,52) = 2.9, p = 0.06$, ANOVA); the psychotic
407 group had greater disconfirmatory updating than the non-clinical controls
408 ($p(adj) = 0.049$) but not the clinical controls ($p(adj) = 0.4$). There was no

409 significant difference in initial certainty across the groups ($F(2,52) = 0.9, p = 0.4$,
410 ANOVA). Differences in final certainty were no longer significant ($F(2,52) = 2.8, p$
411 $= 0.07$, ANOVA); the biggest difference was the non-clinical controls' final
412 certainty which was numerically higher than the clinical controls' ($p(adj) =$
413 0.057).

414 There were negative correlations between initial certainty and
415 disconfirmatory updating at both baseline ($\rho = -0.41, p = 0.00015$) and follow-up
416 ($\rho = -0.41, p = 0.002$), but not between final certainty and the other two
417 measures ($p > 0.1$ in all four comparisons).

418

419 **Behavioural results: dataset 2**

420

421 The mean responses of subjects in each group are plotted in Figure 2B. There
422 was a significant increase in disconfirmatory updating in Scz compared with
423 controls ($t(88.6) = 2.1, p = 0.04$, Welch's t -test). There was mixed evidence for a
424 difference in initial certainty between Scz and controls: Scz were more certain
425 after the first bead in sequences A and B but not C or D (Figure 2 and Table 2),
426 but the difference in mean initial certainty fell short of statistical significance
427 ($t(110) = -1.9, p = 0.059$, Cohen's $d = 0.32$, Welch's t -test). Final certainty was
428 only assessed in sequences A and D (B and C contained two changes of colour in
429 the last three beads): in both sequences, Scz were less certain than controls
430 (sequence A: $t(80.1) = 3.0, p = 0.004$, sequence D: $t(85.5) = 3.4, p = 0.001$, Welch's
431 t -tests).

432 Initial certainty and disconfirmatory updating negatively correlated
433 within both Scz ($\rho = -0.46, p = 0.0003$) and control ($\rho = -0.57, p = 10^{-11}$) groups.
434 Final certainty did not correlate with either measure in either group ($p > 0.4$ in
435 four comparisons).

436

437 **Modelling results: dataset 1**

438

439 Model selection results for the three groups analysed separately at both baseline
440 and follow-up are plotted in Figure 6 (columns 1, 2, 4 and 5); the probability of
441 each model being best for any given subject is shown in the left panel, and the
442 probability of each model being the best overall is shown in the right panel.

443 Model 6 is the clear winner at each time point, although a minority of psychotic
444 and clinical controls are best fit by Model 4.

445 Model 6's parameter distributions are shown in Figure 7; they are
446 skewed, hence non-parametric tests were used to determine group differences
447 (full details in Table 3; $p(adj)$ refers to the adjusted p value of Dunn's *post hoc*
448 test). At baseline there were large group differences in belief instability κ_1
449 ($\chi^2(2, n=80) = 9.64, p = 0.008, \eta^2 = 0.12$, Kruskal-Wallis' one-way ANOVA on
450 ranks) and response stochasticity ν ($\chi^2(2, n=80) = 11.9, p = 0.003, \eta^2 = 0.15$) but
451 not in $\sigma_2^{(0)}$ or ω . There were statistically significant differences in κ_1 between the
452 non-clinical controls and both the psychotic group ($p(adj) = 0.01$, Dunn's test)
453 and the clinical control group ($p(adj) = 0.01$), but not between the latter two
454 groups ($p(adj) = 0.4$). Similarly, there were statistically significant differences in
455 ν between the non-clinical controls and both the psychotic group ($p(adj) = 0.002$,

456 Dunn's test) and the clinical control group ($p(adj) = 0.01$), but not between the
457 latter two groups ($p(adj) = 0.3$).

458 At follow-up, there were still large group differences in κ_1 ($\chi^2(2,n=55) =$
459 $8.0, p = 0.02, \eta^2 = 0.15$, Kruskal-Wallis' one-way ANOVA on ranks) and ν
460 ($\chi^2(2,n=55) = 8.5, p = 0.01, \eta^2 = 0.16$) but not in $\sigma_2^{(0)}$ or ω . There was a significant
461 difference in κ_1 between the psychotic and non-clinical control groups ($p(adj) =$
462 0.007 , Dunn's test) but not the clinical and non-clinical control groups ($p(adj) =$
463 0.1); ν remained significantly different between the non-clinical controls and
464 both the psychotic group ($p(adj) = 0.01$, Dunn's test) and now also between the
465 psychotic and clinical control groups ($p(adj) = 0.01$), but not between the clinical
466 and non-clinical controls ($p(adj) = 0.5$).

467 We explored whether group differences in κ_1 or ν at baseline and follow
468 up might be ascribable to IQ (Quick Test score (Ammons and Ammons, 1962)),
469 as the groups' IQ scores were not equivalent (Table 1). Including both IQ and
470 group status within one regression model is an unsound method of testing for
471 confounding by IQ because group and IQ are clearly not independent here (Miller
472 and Chapman, 2001), so we tested for relationships between the parameters and
473 IQ separately within each group at each time point. No relationships reached
474 statistical significance (all $p > 0.1$), the closest being a trend between κ_1 and IQ in
475 non-clinical controls only ($r = -0.30, p = 0.08$); nevertheless, given the smaller
476 group sizes and larger between- versus within-group variances, it remains
477 plausible that IQ differences contribute to group parameter differences.

478 We tested whether κ_1 or ν at baseline related to delusion-proneness
479 (Peters Delusion Inventory score) across all groups, after first excluding any
480 interaction between PDI and group; PDI significantly correlated with ν ($F(1,67) =$

481 7.1, $p = 0.01$, ANCOVA) but not κ_1 ($F(1,67) = 3.2$, $p = 0.079$, ANCOVA). We tested
482 whether κ_1 or ν at baseline was correlated with any particular subgroup of
483 symptoms (measured using the Manchester Scale (Krawiecka et al., 1977)) in
484 both clinical groups only, using the regression models κ_1 [or ν] \sim const +
485 ν_1 *MSaffective + ν_2 *MSpositive + ν_3 *MSnegative: none of the models were
486 significant, however (all $p > 0.1$).

487 At baseline, there was no evidence of a correlation between κ_1 and
488 antipsychotic medication dose ($p = 0.3$), but the correlation between ν and
489 medication dose approached significance ($\rho = -0.4$, $p = 0.067$).

490 We tested for correlations between the Model 6 parameters (Spearman's
491 ρ was used where distributions were not parametric): κ_1 and ν were negatively
492 correlated both at baseline ($\rho = -0.38$, $p = 0.0004$) and at follow up ($\rho = -0.52$, $p =$
493 0.0001), as were κ_1 and ω at baseline ($\rho = -0.47$, $p = 10^{-5}$) and follow up ($\rho = -$
494 0.53 , $p = 10^{-5}$). In estimating the parameters from *simulated* data, the only
495 correlation present in both simulations (indicating some consistent trading-off
496 between these parameters during estimation) was between κ_1 and ω , with $r = -$
497 0.5 in each case. This is not surprising, as both κ_1 and ω affect updating to new
498 information throughout the sequence (unlike $\sigma_2^{(0)}$) in a deterministic way (unlike
499 ν). Nevertheless, κ_1 was estimated very reliably in the first simulation (Figure 5,
500 top row) and with reasonable accuracy in the second (Figure 5, bottom row), so
501 we are confident that the group differences in κ_1 are genuine. The correlations of
502 $\rho \approx -0.5$ between ω and κ_1 in dataset 1 are unlikely to be reliable, however.

503

504 **Modelling results: dataset 2**

505

506 We tested the same six models and performed Bayesian model selection as
507 before. As in dataset 1, the winning model was Model 6 overall and in each group
508 separately (Figure 6), although in the Scz group a minority were best captured
509 by Model 4. Model 6's parameter distributions are shown in Figure 8; they are
510 skewed, so non-parametric tests were used (full details in Table 3).

511 As in dataset 1, belief instability κ_1 was significantly higher in Scz than in
512 controls ($Z = -5.6$, $p = 10^{-8}$, Mann-Whitney U test) with a medium-to-large effect
513 size ($r = 0.43$); also response stochasticity ν was lower in Scz than in controls (Z
514 $= 3.9$, $p = 0.0001$, $r = 0.3$, Mann-Whitney U test), as was initial belief variance $\sigma_2^{(0)}$
515 ($Z = 3.1$, $p = 0.002$, $r = 0.24$, Mann-Whitney U test). There were no statistically
516 significant group differences in evolution rate ω . See Figures 6 and 7 for
517 examples of model fits in subjects with lower κ_1 values (two controls in Figure 9)
518 and higher κ_1 values (two Scz subjects in Figure 10); each figure also illustrates
519 the effects of lower and higher ω values (in the top and bottom rows
520 respectively). We repeated the analysis using a subset of the controls ($n=60$) that
521 were better matched in age and sex, as the original control group was younger
522 and more female than the patient group (Table 1). The group differences in κ_1
523 and ν were unchanged in this analysis ($Z = -4.1$, $p = 0.00004$; $Z = 3.4$, $p = 0.0007$
524 respectively, Mann-Whitney U tests), but that in $\sigma_2^{(0)}$ was no longer significant (Z
525 $= 1.9$, $p = 0.056$, Mann-Whitney U test).

526 Although IQ (National Adult Reading Test score (Nelson, 1982)) was
527 evenly matched in these groups, working memory (Letter Number Sequencing
528 score (Wechsler, 1997)) was lower in Scz than in controls (see Table 1). We

529 explored whether the group parameter differences might be related to working
530 memory, by testing for correlations between κ_1 or ν and working memory in each
531 group separately (Miller and Chapman, 2001): none were statistically significant
532 (all $p > 0.1$). We also tested for relationships between κ_1 or ν and IQ (NART) in
533 each group: ν and IQ (NART) were correlated in Scz ($r = 0.33, p = 0.014$), but no
534 other relationships were significant (all $p > 0.1$).

535 We tested whether κ_1 or ν related to schizotypy (Schizotypal Personality
536 Questionnaire score) across all groups but neither did so (both $p = 0.4$, ANCOVA).
537 We tested whether κ_1 or ν were predicted by any particular subgroup of
538 symptoms (measured using the Positive and Negative Symptom Scale (Kay et al.,
539 1987)) in the Scz group only, using the regression model κ_1 [or ν] \sim const +
540 ν_1 *PANSSgeneral + ν_2 *PANSSpositive + ν_3 *PANSSnegative: the κ_1 model was not
541 significant ($F = 0.9, p = 0.4$), but ν was weakly predicted by negative symptoms
542 (overall $F = 2.76, p = 0.051$; for $\nu_3, t = -2.1, p = 0.04$). We had no record of
543 medication dose in dataset 2.

544 We tested for correlations between the Model 6 parameters: as in dataset
545 1, κ_1 and ν were negatively correlated (Figure 8; $\rho = -0.35, p = 10^{-6}$), but unlike
546 dataset 1, the only other statistically significant correlation was between κ_1 and
547 $\sigma_2^{(0)}$ ($\rho = -0.54, p = 10^{-13}$). There was a correlation of $r = -0.2$ between κ_1 and ν in
548 the data simulated from modal Scz parameter values (Figure 5, bottom row), but
549 no correlation in the first. This implies that the consistent correlations between
550 these parameters of $\rho = -0.38, \rho = -0.52$ (dataset 1 baseline and follow-up) and ρ
551 $= -0.35$ (dataset 2) are unlikely to be just estimation artefacts. The only other
552 correlation between parameters in the simulated data was between $\sigma_2^{(0)}$ and κ_1 ,

553 of $r = -0.25$, in the first simulation only. These parameters were correlated in
554 dataset 2 but not dataset 1.

555 **Discussion**

556 Scz tend to update their beliefs more to unexpected information and less to
557 consistent information, compared to controls. We have replicated these
558 behavioural effects, and demonstrated a computational basis for them that is
559 informed by the unstable attractor hypothesis of schizophrenia. In
560 computational models of two 'beads task' datasets, Scz had consistently greater
561 belief instability (κ_I) and response stochasticity (ν) than controls, as the unstable
562 attractor hypothesis predicts. Furthermore, ν correlated with κ_I in all three
563 experiments, supporting the idea that ν is measuring a stochasticity that is
564 related to κ_I by an underlying neurobiological process, rather than simply an
565 unmodelled effect.

566 These findings are important because they connect numerous reasoning
567 biases previously found in Scz – e.g. a disconfirmatory bias (Garety et al., 1991;
568 Fear and Healy, 1997; Young and Bentall, 1997; Peters and Garety, 2006),
569 increased initial certainty (Peters and Garety, 2006), and decreased final
570 certainty (Horga, in preparation) – and its associated stochasticity in responding
571 (Moutoussis et al., 2011; Schlagenhauf et al., 2013) to model parameters that
572 describe how belief updating in cortex could be perturbed by unstable attractor
573 states due to NMDA (or dopamine 1) receptor hypofunction (Figure 1).

574 The unique features of Model 6 that make attractor dynamics a
575 compelling neurobiological explanation for its dominance are both Scz and
576 controls' non-linearities in belief updating to confirmatory versus

577 disconfirmatory evidence. The Scz group updated its beliefs (sometimes much)
578 more to disconfirmatory than confirmatory evidence – particularly at points of
579 relative certainty about the jar – and the controls were the opposite. Models with
580 uniformly high or low learning rates cannot reproduce these effects; and adding
581 high- or low-level (sensory) uncertainty to a hierarchical model would lead to
582 uniformly high or low learning rates respectively. Although Models 3 and 4 do
583 show differential updating to confirmatory vs disconfirmatory evidence, this
584 results in beliefs in either jar hovering around 0.5 (as in Figure 4, top left) rather
585 than making large updates from belief in one jar to the other (as when $\kappa_l =$
586 $\exp(1.2)$: Figure 4, bottom left). Furthermore, degraded neuronal ensemble firing
587 (consistent with unstable attractor states) has recently been shown to be
588 common to two different mouse models of schizophrenia (Hamm et al., 2017).

589 In dataset 1, belief instability κ_l and response stochasticity ν were also
590 significantly different between the clinical (mood disorder) and non-clinical
591 control groups when the former were unwell, but not at follow-up, whereas the
592 differences between the psychotic group and non-clinical controls persisted. This
593 indicates that the same computational parameters can be perturbed in either a
594 trait- or state-like manner, perhaps by different mechanisms. It seems unlikely
595 that these parameter changes simply reflect a lack of engagement with the task
596 in clinical groups (especially when unwell), because the consistent changes in κ_l
597 – with which the changes in ν consistently correlate – reflect specific patterns of
598 belief updating.

599 **Parameter relationships with cognition and symptoms**

600

601 Neither κ_1 nor ν showed significant relationships with IQ (in dataset 1) or
602 working memory (in dataset 2) within the groups, giving some indication that
603 the group differences in these cognitive measures were unlikely to be the main
604 drivers of group differences in the parameters. Nevertheless, aside from the
605 correlation between response stochasticity ν and IQ in dataset 2, it is perhaps
606 surprising that there weren't more relationships between κ_1 or ν and cognitive
607 measures in Scz, given it is likely that abnormal prefrontal dynamics have
608 profound effects on all these variables. We may have lacked power to detect
609 them – though dataset 2 had 80% power to detect a correlation of 0.33 – or
610 perhaps different prefrontal regions contribute to working memory, IQ and
611 belief updating.

612 One might also question why there were no strong relationships between
613 κ_1 or ν and positive or negative symptom domains (negative symptoms were
614 weakly associated with ν in dataset 2 only). Again, power may have been an
615 issue, although note that across all subjects in dataset 1, response stochasticity ν
616 was associated with PDI score even after including group in the model, indicating
617 a potential relationship with delusions, but not with the broader concept of
618 schizotypy (assessed in dataset 2). It is also likely that other pathological factors
619 contribute to symptoms, beyond those measured here (e.g. striatal dopamine
620 availability and positive symptoms). Of note, two other computational studies
621 demonstrating clear working memory parameter differences between Scz and
622 controls also failed to detect any relationship between those parameters and
623 symptom domains (Collins et al., 2014, 2017). Both their and our Scz groups

624 were taking antipsychotic medication, which is also likely to weaken correlations
625 of parameters to positive symptoms.

626 Although replicated numerous times in the beads task, a ‘disconfirmatory
627 bias’ is perhaps surprising in Scz, given one might expect delusional subjects to
628 show a bias *against* disconfirmatory evidence (as indeed they do in tasks
629 involving scenario interpretation (Woodward et al., 2006)). In fact, the
630 disconfirmatory bias is misleadingly named, as Scz make large shifts in beliefs
631 both away from *and back towards* the current hypothesis (there are numerous
632 examples in both datasets in Figure 2). This pronounced switching behaviour in
633 the beads task is likely to illustrate a more fundamental instability of cognition
634 and prefrontal dynamics in Scz, rather than being related to delusions
635 specifically; indeed, the latter may be an attempt to remedy the former.

636 It is interesting that non-clinical controls’ data were also best fit by Model
637 6 in both datasets, implying that even healthy subjects show some asymmetry in
638 their belief updating to expected versus unexpected evidence. Most non-clinical
639 control subjects had $\kappa_1 < 1$, i.e. reduced updating to changing evidence.

640 **Related modelling studies**

641 How do these findings relate to other computational modelling work in
642 Scz? A study of unmedicated, mainly first episode Scz performing a reversal
643 learning task (Schlagenhauf et al., 2013) also demonstrated an increased
644 tendency to switch that was not accounted for by reward sensitivity (which
645 would be affected by more stochastic behaviour), and increased switching also
646 occurs in chronic Scz (Waltz et al., 2013), although not always (Pantelis et al.,
647 1999).

648 Two recent studies of similar tasks in Scz populations have also
649 demonstrated evidence of non-linear belief updating. (Jardri et al., 2017) showed
650 that the Scz group on average “overcount” the likelihood in a single belief update;
651 an effect they attribute to reverberating cortical message-passing, but which
652 could also be due to the belief instability shown by Model 6. (Stuke et al., 2017)
653 showed in a very similar task that all subjects showed evidence of non-linear
654 updating, but the Scz group updated more than controls to “irrelevant
655 information” (i.e. disconfirmatory evidence). Some differences between their
656 model and ours are that they did not estimate response stochasticity in their
657 subjects (neither did (Jardri et al., 2017), and their ‘non-linearity’ parameter was
658 bounded by linear updating on one side, roughly equivalent to belief instability
659 κ_1 being constrained to being <1 in our model, whereas we have shown (as in
660 (Jardri et al., 2017) that Scz belief updating is often beyond this bound (Figure 7),
661 and more stochastic. Conversely, (Moutoussis et al., 2011) demonstrated
662 increased response stochasticity in acutely psychotic subjects, but did not test
663 for differences in belief updating.

664 The extent to which a loss of belief stability in Scz is apparent depends
665 critically on the strength (precision) of incoming sensory evidence relative to the
666 current belief (prior): if the former is less precise, no belief switching may occur,
667 and instead the percept may be weighted towards the prior. In the beads task,
668 sensory evidence (i.e., the colour of the bead drawn) is unambiguous, but a task
669 using very imprecise auditory sensory evidence (Powers et al., 2017)
670 demonstrated some interesting heterogeneity in Scz: non-hallucinating Scz
671 showed greater belief updating relative to controls, while in hallucinating Scz,

672 percepts were driven by prior expectations, leading to a reduction in the
673 updating of their beliefs (relative to controls).

674 Further evidence for heterogeneity in Scz is that those with delusions
675 have greater certainty about the hypothesis that matches the evidence at every
676 stage (Speechley et al., 2010), unlike the reduced final certainty we observed in
677 Scz in dataset 2. On the other hand, Scz with high negative symptoms have
678 difficulty choosing the most rewarding option very consistently (Gold et al.,
679 2012), which may reflect a lack of certainty about its value. We lacked sufficient
680 power to detect differences between Scz with exclusively high positive or
681 negative symptoms, however.

682 **Limitations**

683 Each of our datasets contains some limitations of the beads task that are
684 addressed by the other. Dataset 1 did not include a memory aid or measure
685 working memory, but dataset 2 did both, and dataset 2 also matched IQ across
686 groups much better than dataset 1; dataset 2 used a Likert scale for responding
687 and so could potentially exaggerate small changes in belief updating, but dataset
688 1 used a continuous measure; dataset 2 only tested stable outpatients, but
689 dataset 1 tested more unwell inpatients and retested them once they were
690 better. The main limitation common to both datasets is that all subjects with
691 psychotic diagnoses were taking antipsychotic medication when tested. Although
692 the correlation between v and medication dose was almost significant in dataset
693 1, this relationship seems likely to be driven by illness severity rather than
694 medication itself. Dopamine 2 receptor antagonists seem to both reduce
695 overconfidence in probabilistic reasoning (Andreou et al., 2014), and also

696 reduce motor response variability (Galea et al., 2013) and so if anything likely
697 reduce our group differences.

698 **Conclusion**

699 In conclusion, we have shown that Scz subjects in two independent beads
700 task datasets have consistent differences in two parameters of a belief updating
701 model that attempts to reproduce consequences of attractor network instability.
702 Note that this study was designed to link patterns of inferences to model
703 parameters that (do or don't) mimic the effects of abnormal attractor states on
704 belief updating. The HGF itself does not contain attractor states and no relation
705 between its parameters and NMDAR function has hitherto been tested. More
706 detailed spiking network modelling, pharmacological (or other NMDAR)
707 manipulations and imaging are required in future to understand how
708 neuromodulatory function in both pyramidal cells and inhibitory interneurons
709 contributes to real attractor dynamics and probabilistic inference, and to seek
710 empirical evidence for a correspondence between the stability of network states
711 and the stability of its inferences (especially in schizophrenia). This work
712 underscores the importance of relating psychological biases to their underlying
713 computational mechanisms, and thence (in future) to the constraints – e.g. the
714 hypofunction of NMDARs – that neurobiology imposes on these mechanisms.
715

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721

722

723

724 **Conflicts of Interest**

725 No authors have any biomedical financial interests or potential conflicts of

726 interest.

727

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896 **Figure Legends**

897

898 **Figure 1: Effects of attractor network dynamics on belief updating**

899

900 This schematic illustrates the energy landscapes of two Hopfield-type networks
901 each with two basins of attraction. The continuous black line depicts a normal
902 network whose basins of attraction are relatively deep. The dotted black line
903 depicts the effect of NMDAR (or cortical dopamine 1 receptor (Durstewitz and
904 Seamans, 2008)) hypofunction (Abi-Saab et al., 1998; Javitt et al., 2012) on the
905 energy landscape: the attractor basins become more shallow. We assume that
906 Basins A and B correspond to different inferences about (hidden) states in the
907 world, e.g. one jar or another being the source of beads in the beads task. The
908 dots correspond to the networks' representations of either control or Scz
909 subjects' beliefs about these hidden states. Such networks are highly reminiscent
910 of Hopfield networks with two stored representations – in this case, the
911 representations correspond to inferences about hidden states, rather than
912 memories. The arrows depict the changes in network states resulting from
913 sensory evidence for (solid arrows) or against (dashed arrows) the current
914 inference. When the attractor basin is shallower, it is harder for supportive
915 evidence to stabilise the current state much further, but it is easier for
916 contradictory evidence – or just stochastic neuronal firing – to shift the current
917 network state towards an alternative state. These changes in network dynamics
918 may also be reflected in the inferences the network computes – i.e. easier
919 switching between attractor basins may correspond to easier switching between

920 beliefs – although this is yet to be demonstrated experimentally. NMDAR
921 hypofunction could contribute to an increased tendency to switch between
922 beliefs and increased stochasticity in responding in several ways (Rolls et al.,
923 2008): i) by reducing inhibitory interneuron activity, via weakened NMDAR
924 synapses from pyramidal cells to interneurons, such that other attractor states
925 are less suppressed when one is active (a spiking network model has shown that
926 this leads to more rapid initial belief updating in perceptual tasks (Lam et al.,
927 2017)), ii) by reducing pyramidal cell activity, via weakened recurrent NMDAR
928 synapses on pyramidal cells, such that attractor states are harder to sustain, and
929 iii) by reducing the NMDAR time constant, making states more vulnerable to
930 random fluctuations in neural activity. See also similar schematics elsewhere
931 (Durstewitz and Seamans, 2008; Rolls et al., 2008).

932

933 **Figure 2: Beads task schematic and group average confidence ratings in**
934 **Datasets 1 and 2.**

935

936 The bottom right panel is an illustrative schematic of the beads task: two jars
937 containing opposite proportions of beads are concealed from view and a subject
938 is asked to rate the probability of either jar being the source of a sequence of
939 beads he/she is viewing (after each bead in turn). The top left panel shows the
940 mean (\pm standard error) confidence ratings in the blue jar over the 10 bead
941 sequence averaged across each group at baseline in dataset 1. The bottom left
942 panel shows the same quantities at follow-up in dataset 1. The top right panel
943 shows these quantities in four 10 bead sequences concatenated together (they
944 were presented to the subjects separately during testing) in dataset 2.

945

946 **Figure 3: The structure of the Hierarchical Gaussian Filter (Model 6) and**947 **some simulated data**

948

949 In the upper left panel, the evolution of μ_2 , the posterior estimate of tendency x_2

950 towards the blue (positive) or red (negative) jar, is plotted over two

951 concatenated series of 10 trials (the first two in dataset 2). The estimate of the

952 tendency on trial $k+1$, $\mu_2^{(k+1)}$, is selected from a Gaussian distribution with mean953 $\mu_2^{(k)}$ (blue line) and variance $\sigma_2^{(k)} + \exp(\omega)$ (blue shading). ω is a static source of954 variance at this level. The initial variance $\sigma_2^{(0)}$ (along with ω) affects the size of

955 initial updates, so we estimated this parameter (which is often fixed). The beads

956 seen by the subjects, $u^{(k)}$ (blue and red dots) and the response model are957 illustrated in the bottom left panel. The response model maps from $\hat{\mu}_1^{(k+1)}$ 958 (purple line) – the prediction of x_1 on the next trial, which is a sigmoid function s 959 of $\mu_2^{(k)}$ (or of $(\kappa_1 \mu_2^{(k)})$ in Models 5 and 6) – to $y^{(k)}$, the subject's indicated

960 estimate of the probability the jar is blue (green dots). Variation in this mapping

961 is modelled as the precision ν of a beta distribution.

962 The right panel is a schematic representation of the generative model in Models

963 5 and 6 (i.e. including κ_1). The black arrows denote the probabilistic network on964 trial k ; the grey arrows denote the network at other points in time. The

965 perceptual model lies above the dotted arrows, and the response model below

966 them. The shaded circles are known quantities, and the parameters and states in

967 unshaded circles are estimated. The dotted line represents the result of an

968 inferential process (the response model builds on a perceptual model inference);
 969 the solid lines are generative processes.

970

971 **Figure 4: Simulated data illustrating the effects of φ (Models 3 and 4)**
 972 **and κ_1 (Model 5 and 6) on inference**

973

974 This figure illustrates the effects of φ (used in Models 3 and 4) and κ_1 (used in
 975 Models 5 and 6) on inference. Both panels show simulated perceptual model
 976 predictions in the same format as before, with $\sigma_2^{(0)}$ and ω set to their previous
 977 values – hence the purple line in these plots is identical to that in Figure 3. The
 978 second level and simulated responses y have been omitted for clarity.

979 Upper left panel: Simulations of a perceptual model incorporating an
 980 autoregressive order (1) process at the second level, using three different values
 981 of AR(1) parameter φ : 0, 0.2 and 0.8. The estimate of the tendency on trial $k+1$,
 982 $\mu_2^{(k+1)}$, is selected from a Gaussian distribution with mean $\mu_2^{(k)} + \varphi(m - \mu_2^{(k)})$
 983 and variance $\sigma_2^{(k)} + \exp(\omega)$. Over time, μ_2 is therefore attracted towards level m
 984 (fixed to 0, i.e. at $\sigma(\mu_2) = 0.5$) at a rate determined by φ . In effect, this gives the
 985 model a ‘disconfirmatory bias’, such that as φ increases, $\sigma(\mu_2)$ is pulled further
 986 away from a belief in either jar, and towards 0.5 (maximum uncertainty about
 987 the jars).

988 Lower left panel: Simulations of a perceptual model using four different values of
 989 scaling factor κ_1 , which alters the sigmoid transformation: $\hat{\mu}_1^{(k+1)} = s(\kappa_1 \cdot \mu_2^{(k)})$.
 990 When $\kappa_1 > \exp(0)$ updating is greater to unexpected evidence and lower to
 991 consistent evidence; when $\kappa_1 < \exp(0)$ the reverse is true. The red and brown

992 lines ($\kappa_l > \exp(0)$) illustrate the effects of increasingly unstable attractor
993 networks, i.e. switching between states (jars) becomes more likely (a
994 concomitant increase in vulnerability to noise, i.e. response stochasticity, is not
995 shown). The green line ($\kappa_l = \exp(-1)$) illustrates slower updating around $\hat{\mu}_1 = 0.5$,
996 as was found in controls. κ_l permits a greater range of updating patterns than φ
997 (the green and brown trajectories in the lower panel cannot be produced by
998 Model 4) which may be why Model 6 can fit both controls and Scz groups well.
999 Middle panel: This plot shows the effects of κ_l on belief updating, as a function of
1000 the initial belief $\hat{\mu}_1$ ($\sigma_2^{(0)}$ and ω were set to 1.5 and -1 respectively, as in Figure 5;
1001 changing these parameters does not qualitatively alter the effects of κ_l shown
1002 here). For values of $\kappa_l < \exp(0)=1$ (bottom three curves) and initial beliefs to the
1003 left of these curves' maxima (i.e. that the jar is probably red), relatively small
1004 increases in $\hat{\mu}_1$ are made if one blue bead ($u = 1$) is observed, such that the
1005 subject still believes the jar is most likely red. For values of $\kappa_l > \exp(0.5)$ (top two
1006 curves), observing one blue bead causes such a large update for all but the most
1007 certain initial beliefs in a red jar that the subject's posterior belief is that the jar
1008 is probably blue. These subjects' beliefs are no longer stable, but neither can they
1009 reach certainty: only tiny updates towards 1 are possible for $\hat{\mu}_1 > 0.8$.
1010 Right panel: This plot illustrates the average absolute shifts in beliefs on
1011 observing beads of either colour. This 'vulnerability to updating' is highly
1012 reminiscent of the 'energy state' of a neural network model (schematically
1013 illustrated in Figure 1) - i.e. in low energy states, less updating is expected. The
1014 effect of increasing κ_l is to convert confident beliefs about the jar (near 0 and 1)
1015 from low to high 'energy states', i.e. to make them much more unstable.
1016

1017 **Figure 5: Recovery of model parameters from simulated data**

1018

1019 200 datasets were simulated using Model 6; 100 using modal parameter values
1020 for the control group (dataset 2) and 100 using modal values for the Scz group
1021 (also dataset 2) – the values are indicated using red lines. Both used settings of
1022 $\sigma_2^{(0)} = 1.5$, $\omega = -1$. The control group used $\kappa_1 = 0.37$ (i.e. $\exp(-1)$) and $\nu = \exp(3)$.
1023 The Scz group used $\kappa_1 = 2.7$ (i.e. $\exp(1)$) and $\nu = \exp(2)$. Histograms depicting the
1024 parameter estimates from model inversion using the same priors as were
1025 employed in the main analysis are shown above: the modal control and Scz
1026 simulation results are in the upper and lower rows respectively.

1027

1028 **Figure 6: Bayesian model selection results for both datasets.**

1029

1030 The left panel depicts the protected exceedance probabilities for the six models in
1031 each group in each dataset. The protected exceedance probability is the
1032 probability a particular model is more likely than any other tested model, above
1033 and beyond chance, given the group data (Rigoux et al., 2014). Model 6 wins in all
1034 groups in both datasets (upper row: controls, middle row: Scz, bottom row:
1035 clinical controls).

1036 The right panel depicts the model likelihoods for the six models in each group in
1037 each dataset. The model likelihood is the probability of that model being the best
1038 for any randomly selected subject (Stephan et al., 2009). Model 4 is a clear runner-
1039 up in the psychotic (Scz) and clinical control groups at baseline in dataset 1, and
1040 in the Scz group in dataset 2.

1041

1042 **Figure 7: Probability density plots for Model 6 parameters in dataset 1.**

1043

1044 The distributions of parameter values for $\sigma_2^{(0)}$, ω , $\log(\nu)$ and $\log(\kappa_1)$ are plotted
1045 for dataset 1 at baseline (upper row) and dataset 1 at follow-up (lower row). The
1046 symbols denote significant group differences: § between non-clinical controls
1047 and clinical controls, * between non-clinical controls and Scz, † between Scz and
1048 clinical controls. Please see the text for the details of all statistical comparisons.

1049

1050 **Figure 8: Model 6 parameters in dataset 2 – distributions and correlation**

1051

1052 Upper panel: The distributions of parameter values for $\sigma_2^{(0)}$, ω , $\log(\nu)$ and $\log(\kappa_1)$
1053 are plotted for dataset 2. The * symbol denotes significant group differences
1054 between the Scz group and non-clinical control subgroup (well-matched in age
1055 and sex); the group difference in $\sigma_2^{(0)}$ is not indicated because it was non-
1056 significant ($p=0.056$) in the well-matched comparison. Please see the text for the
1057 details of all statistical comparisons.

1058 Lower panel: The significant correlation between $\log(\nu)$ and $\log(\kappa_1)$ in dataset 2
1059 is plotted, with controls' parameters in black and Scz in red. Similar correlations
1060 were also found in dataset 1 at both time points (see text).

1061

1062 **Figure 9: Responses and model fits for two control subjects**

1063

1064 These plots show two control subjects' responses to four ten-bead sequences
1065 concatenated together, in the same format as Figure 3 (but without the second
1066 level, due to space constraints); in the latter two sequences blue and red were

1067 swapped around for model-fitting purposes. Each plot shows $u^{(k)}$ – the beads
1068 seen by the subjects on trials $k = 1, \dots, 10$ (blue and red dots), y – the subject’s
1069 (Likert scale) response about the probability the jar is blue (green dots), and
1070 $\hat{\mu}_1^{(k+1)}$ – the model’s estimate of the subject’s prediction the jar is blue (purple
1071 line). The parameter estimates for each subject are shown above their graphs.
1072 These subjects have fairly similar initial variance $\sigma_2^{(0)}$, (inverse) response
1073 stochasticity ν , and instability factor κ_1 . Subject 18 in the upper panel has a much
1074 lower overall evolution rate ω than Subject 67 in the lower panel, therefore
1075 Subject 18 never reaches certainty about either jar, and makes relatively small
1076 changes to her beliefs in response to beads of varying colours. Both subjects have
1077 a low κ_1 , and so they make relatively small adjustments to their beliefs following
1078 unexpected evidence (this behaviour can best be captured by the models
1079 containing κ_1 – see Figure 4). Subject 18’s responses are very close to those
1080 predicted by the model, and this is reflected in her relatively high value of ν .

1081

1082 **Figure 10: Responses and model fits for two Scz subjects**

1083

1084 These plots show two Scz subjects’ responses to four ten-bead sequences in the
1085 same format as Figure 9. These subjects have similar evolution rate ω to the
1086 control subjects in Figure 9, but they both have a much higher κ_1 , meaning that
1087 they make much greater changes to their beliefs when presented with
1088 unexpected evidence, but do not reach certainty when faced with consistent
1089 evidence. Subject 122 (lower panel) has a slightly higher evolution rate ω than
1090 Subject 145 (upper panel), and so his switching between jars is even more
1091 pronounced. These subjects also have slightly lower (inverse) response

1092 stochasticity v than the control subjects in Figure 9, and so their responses tend

1093 to be further from the model predictions.

1094

1095

1096

Dataset 1							Dataset 2			
	Non-clinical controls t1	Non-clinical controls t2	Clinical controls t1	Clinical controls t2	Psychotic t1	Psychotic t2		Controls (all)	Scz	Controls (subset)
N	35	20	22	18	23	17	N	111	56	60
Age ^a	27.77 (6.74)	27.9 (6.37)	40.91 (13.57)	40.1 (13)	31.22 (7.28)	29.9 (7.83)	Age	32.8 (11.5)	45.3 (8.8)	39.5 (11.4)
Gender	18 M, 17 F	12 M, 8 F	11 M, 11 F	8 M, 10 F	21 M, 2 F	17 M, 0 F	Gender	45 M, 66 F	38 M, 18 F	40 M, 20 F
Cognitive measures										
IQ ^b	107.5 (11.6)	108.6 (10.3)	97.4 (13.8)	99.8 (10.2)	88.1 (12.7)	87.8 (14.2)	NART ^a	112 (6.9)	109 (8.2)	112 (7.5)
							Working memory (LNS) ^b	16.2 (2.8)	10.3 (4.2)	16.4 (2.7)
Delusion proneness							Schizotypy			
PDI (total) ^c	54.6 (43.1)	43.6 (42.5)	87.1 (55.2)	64.3 (57.3)	138.1 (74.2)	96.7 (42.6)	SPQ, cognitive	2.8 (1.9)	4.0 (2.6)	3.1(2)
DSSI ^d	2.3 (4.9)	2.9 (5.3)	4.8 (4.5)	4.5 (5.6)	15.2 (6.3)	8.1 (6.6)	SPQ, interpers	3.2 (2.2)	5.3 (2.6)	3.2 (2.2)

							SPQ, disorg	2.1 (1.7)	2.7 (1.9)	1.9 (1.8)
							SPQ, total ^c	8.2 (1.3)	12 (5.3)	8.2 (4.4)
Diagnosis/ Symptoms										
Diagnoses	-	-	16 Depression, 3 anxiety & depression, 3 SAD	12 Depression, 3 anxiety & depression, 3 SAD	18 Scz, 5 bipolar/ schizo- affective	13 Scz, 4 bipolar/ schizo- affective	Diagnoses	-	56 Scz	-
MS affective	-	-	4.6 (1.7)	1.0 (1.2)	1.8 (1.5)	1.5 (1.3)	PANSS, gen	-	32.6 (9.2)	-
MS positive	-	-	0.3 (0.8)	0 (0)	6.0 (2.4)	1.4 (1.7)	PANSS, pos	-	15.9 (5.8)	-
MS negative	-	-	0.7 (1.6)	1.8 (3.19)	1.3 (2.0)	0.9 (1.6)	PANSS, neg	-	15.9 (6.2)	-
MS total ^e	-	-	5.5 (2.6)	2.8 (3.39)	9.1 (3.76)	3.7 (3.9)	PANSS, total	-	64.4 (17.3)	-
Beads task										
Initial certainty (1 bead) ^f	0.58 (0.15)	0.59 (0.12)	0.68 (0.19)	0.63 (0.16)	0.76 (0.17)	0.68 (0.29)	Initial certainty (all, 1 bead) ^d	0.67 (0.13)	0.71 (0.14)	0.68 (0.14)

Initial certainty (3 beads) ^g	0.65 (0.14)	0.67 (0.1)	0.69 (0.15)	0.64 (0.16)	0.78 (0.15)	0.74 (0.15)	Initial certainty (all, 2-3 beads) ^e	0.7 (0.12)	0.71 (0.12)	0.71 (0.13)
Disconfirmatory updating ^h	-0.06 (0.14)	-0.03 (0.13)	-0.19 (0.3)	-0.11 (0.22)	-0.29 (0.33)	-0.2 (0.3)	Disconfirmatory updating (all sequences) ^f	-0.16 (0.17)	-0.23 (0.22)	-0.19 (0.2)
Final certainty ⁱ	0.85 (0.2)	0.94 (0.11)	0.82 (0.16)	0.79 (0.23)	0.88 (0.11)	0.85 (0.23)	Final certainty Sequence A ^g	0.88 (0.16)	0.77 (0.25)	0.86 (0.18)
							Final certainty Sequence D ^h	0.12 (0.18)	0.25 (0.24)	0.16 (0.2)

1097

1098

Table 1: Demographic, psychological and behavioural details of both datasets

1099

1100 Dataset 1 includes measures at both baseline (t1) and follow-up (t2). In dataset 1, verbal IQ was estimated using the Quick Test (Ammons and Ammons, 1962)
 1101 and delusion proneness using the Peters Delusion Inventory, PDI (Peters et al., 1999) and Delusions-Symptoms-States Inventory, DSSI (Foulds and Bedford,
 1102 1975). Symptoms were assessed using the Manchester Scale, MS (Krawiecka et al., 1977). In the tests below, 'Scz' refers to the whole Psychotic group.

1103 Results are given for 'Initial certainty' using both the measure in the original analysis of dataset 1 (Peters and Garety, 2006), the mean response to the first
 1104 three beads ('3 beads') - in dataset 2 this had to be the mean response to the first three beads in sequences B and C and two beads in sequences A and D ('2-
 1105 3 beads') - and using the response to the first bead ('1 bead').

1106

1107

1108 ^a At t1: One-way ANOVA $F(2,77) = 13.9, p = 10^{-5}$. Tukey's HSD: Scz vs Non-clinical controls diff
1109 = 3.45, $p(\text{adj}) = 0.35$; Clinical vs Non-clinical controls diff = 13.1, $p(\text{adj}) = 10^{-5}$; Clinical controls
1110 vs Scz diff = 9.69, $p(\text{adj}) = 0.002$
1111 At t2: One-way ANOVA $F(2,52) = 8.85, p = 0.0005$. Tukey's HSD: Scz vs Non-clinical controls
1112 diff = 1.98, $p(\text{adj}) = 0.8$; Clinical vs Non-clinical controls diff = 12.2, $p(\text{adj}) = 0.0006$; Clinical
1113 controls vs Scz diff = 10.2, $p(\text{adj}) = 0.007$
1114 ^b At t1: One-way ANOVA $F(2,75) = 16.2, p = 10^{-6}$; Tukey's HSD: Scz vs Non-clinical controls diff
1115 = -19.5, $p(\text{adj}) = 10^{-6}$; Clinical vs Non-clinical controls diff = -10.1, $p(\text{adj}) = 0.011$; Clinical
1116 controls vs Scz diff = 9.36, $p(\text{adj}) = 0.043$
1117 At t2: One-way ANOVA $F(2,51) = 14.5, p = 10^{-5}$; Tukey's HSD: Scz vs Non-clinical controls diff
1118 = -20.8, $p(\text{adj}) = 10^{-5}$; Clinical vs Non-clinical controls diff = -8.8, $p(\text{adj}) = 0.057$; Clinical
1119 controls vs Scz diff = 12, $p(\text{adj}) = 0.01$
1120 ^c At t1: One-way ANOVA $F(2,68) = 12.6, p = 0.00002$; Tukey's HSD: Scz vs Non-clinical controls
1121 diff = 83.5, $p(\text{adj}) = 10^{-5}$; Clinical vs Non-clinical controls diff = -32.5, $p(\text{adj}) = 0.094$; Clinical
1122 controls vs Scz diff = -51, $p(\text{adj}) = 0.016$
1123 At t2: One-way ANOVA $F(2,52) = 4, p = 0.024$; Tukey's HSD: Scz vs Non-clinical controls diff =
1124 53.1, $p(\text{adj}) = 0.018$; Clinical vs Non-clinical controls diff = -20.7, $p(\text{adj}) = 0.5$; Clinical controls
1125 vs Scz diff = -32.4, $p(\text{adj}) = 0.22$
1126 ^d At t1: One-way ANOVA $F(2,76) = 43, p = 10^{-13}$; Tukey's HSD: Scz vs Non-clinical controls diff
1127 = 12.9, $p(\text{adj}) = 10^{-10}$; Clinical vs Non-clinical controls diff = 2.52, $p(\text{adj}) = 0.19$; Clinical controls
1128 vs Scz diff = -10.4, $p(\text{adj}) = 10^{-8}$
1129 At t2: One-way ANOVA $F(2,51) = 3.7, p = 0.032$; Tukey's HSD: Scz vs Non-clinical controls diff
1130 = 5.2, $p(\text{adj}) = 0.026$; Clinical vs Non-clinical controls diff = 1.65, $p(\text{adj}) = 0.66$; Clinical controls
1131 vs Scz diff = -3.56, $p(\text{adj}) = 0.18$
1132 ^e At t1: Welch's $t(38.4) = -3.62, p = 0.00086$, Cohen's $d = 1.1$
1133 At t2: Welch's $t(17.8) = -2.55, p = 0.02$, Cohen's $d = 1.0$
1134 ^f At t1: One-way ANOVA $F(2,77) = 8.7, p = 0.0004$; Tukey's HSD: Scz vs Non-clinical controls
1135 diff = 0.18, $p(\text{adj}) = 0.0003$; Clinical vs Non-clinical controls diff = 0.11, $p = 0.06$; Clinical
1136 controls vs Scz diff = -0.08, $p(\text{adj}) = 0.25$
1137 At t2: One-way ANOVA $F(2,52) = 0.9, p = 0.4$
1138 ^g At t1: One-way ANOVA $F(2,77) = 6.2, p = 0.003$; Tukey's HSD: Scz vs Non-clinical controls diff
1139 = -0.14, $p(\text{adj}) = 0.002$; Clinical vs Non-clinical controls diff = 0.04, $p = 0.57$; Clinical controls
1140 vs Scz diff = -0.096, $p(\text{adj}) = 0.074$
1141 At t2: One-way ANOVA $F(2,52) = 2.35, p = 0.11$; Tukey's HSD: Scz vs Non-clinical controls diff
1142 = 0.07, $p(\text{adj}) = 0.28$; Clinical vs Non-clinical controls diff = -0.03, $p = 0.8$; Clinical controls vs
1143 Scz diff = -0.1, $p(\text{adj}) = 0.1$

1144 ^h At t1: One-way ANOVA $F(2,77) = 6, p = 0.004$; Tukey's HSD: Scz vs Non-clinical controls diff
 1145 = $-0.23, p(adj) = 0.003$; Clinical vs Non-clinical controls diff = $-0.14, p = 0.13$; Clinical controls
 1146 vs Scz diff = $0.097, p(adj) = 0.41$

1147 At t2: One-way ANOVA $F(2,52) = 2.9, p = 0.062$; Tukey's HSD: Scz vs Non-clinical controls diff
 1148 = $-0.18, p(adj) = 0.049$; Clinical vs Non-clinical controls diff = $-0.08, p = 0.51$; Clinical controls
 1149 vs Scz diff = $0.098, p(adj) = 0.4$

1150 ⁱ At t1: One-way ANOVA $F(2,77) = 0.71, p = 0.5$

1151 At t2: One-way ANOVA $F(2,52) = 2.79, p = 0.07$; Tukey's HSD: Scz vs Non-clinical controls diff
 1152 = $-0.082, p(adj) = 0.41$; Clinical vs Non-clinical controls diff = $-0.15, p = 0.057$; Clinical controls
 1153 vs Scz diff = $-0.066, p(adj) = 0.57$

1154 As reported previously, there were consistent negative correlations between initial certainty
 1155 (2-3 beads) and disconfirmatory updating in the clinical controls (baseline: $\rho = -0.68, p =$
 1156 0.0005 ; follow-up: $\rho = -0.75, p = 0.0003$) and the non-clinical controls (baseline: $\rho = -0.52, p =$
 1157 0.001 ; follow-up: $\rho = -0.43, p = 0.06$), but not in the psychotic group (baseline: $\rho = -0.30, p =$
 1158 0.17 ; follow-up: $\rho = 0.17, p = 0.5$). There was no consistent correlation between final certainty
 1159 and either of the other two measures at either time point ($p \geq 0.1$ in 11 out of 12 comparisons).
 1160 In dataset 2, IQ was estimated using the National Adult Reading Test, NART (Nelson, 1982)
 1161 and working memory using the Letter Number Sequencing task, LNS, from the Wechsler Adult
 1162 Intelligence Scale-III (Wechsler, 1997). Schizotypy was assessed using the Schizotypal
 1163 Personality Questionnaire, SPQ (Raine, 1991), and symptoms using the Positive and Negative
 1164 Syndrome Scale, PANSS (Kay et al., 1987).

1165 As can be seen in Figure 2 (main text), the Scz group showed greater initial certainty (1 bead)
 1166 in sequences A and B (Welch's $t(94) = 2.8, p = 0.007$, Cohen's $d = 0.47$; Welch's $t(97) = 3, p =$
 1167 0.004 , Cohen's $d = 0.5$, respectively) but not C and D (Welch's $t(87) = 0.5, p = 0.6$, Cohen's $d =$
 1168 0.09 ; Welch's $t(90) = -0.34, p = 0.73$, Cohen's $d = 0.06$, respectively).

1169 ^a Controls (all): Welch's $t(95.1) = 2.27, p = 0.026$, Cohen's $d = 0.38$; Controls (subset): Welch's
 1170 $t(111) = 1.95, p = 0.053$, Cohen's $d = 0.36$

1171 ^b Controls (all): Welch's $t(81) = 9.57, p = 10^{-14}$, Cohen's $d = 1.66$; Controls (subset): Welch's
 1172 $t(93.6) = 9.25, p = 10^{-15}$, Cohen's $d = 1.73$

1173 ^c Controls (all): Welch's $t(92.4) = -4.64, p = 10^{-5}$, Cohen's $d = 0.78$; Controls (subset): Welch's
 1174 $t(107) = -4.19, p = 10^{-5}$, Cohen's $d = 0.78$

1175 ^d Controls (all): Welch's $t(110) = -1.9, p = 0.059$, Cohen's $d = 0.32$; Controls (subset): Welch's
 1176 $t(110) = -1.1, p = 0.28$, Cohen's $d = 0.2$

1177 ^e Controls (all): Welch's $t(109.1) = -0.76, p = 0.45$, Cohen's $d = 0.12$; Controls (subset): Welch's
 1178 $t(113.9) = -0.19, p = 0.85$, Cohen's $d = 0.03$

1179 ^f Controls (all): Welch's $t(88.2) = 2.09, p = 0.04$, Cohen's $d = 0.36$; Controls (subset): Welch's
 1180 $t(110.4) = -0.94, p = 0.35$, Cohen's $d = 0.18$

1181 ^g Controls (all): Welch's $t(80.1) = 2.99, p = 0.0038$, Cohen's $d = 0.56$; Controls (subset): Welch's
 1182 $t(98.7) = 2.18, p = 0.032$, Cohen's $d = 0.41$
 1183 ^h Controls (all): Welch's $t(85.5) = -3.41, p = 0.001$, Cohen's $d = 0.62$; Controls (subset): Welch's
 1184 $t(106) = -2.21, p = 0.029$, Cohen's $d = 0.42$
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 1186
 1187
 1188

Model	Perceptual model parameters (prior mean in native space, prior variance in estimation space)				Response model parameter
	Evolution rate	Initial variance of belief re jars	Disconfirmatory bias	Belief instability	Response stochasticity
1	ω (-2, 16)				ν (exp(4.85), 1)
2	ω (-2, 16)	$\sigma_2^{(0)}$ (0.8, 0.5)			ν (exp(4.85), 1)
3	ω (-2, 16)		ϕ (0.1, 2)		ν (exp(4.85), 1)
4	ω (-2, 16)	$\sigma_2^{(0)}$ (0.8, 0.5)	ϕ (0.1, 2)		ν (exp(4.85), 1)
5	ω (-2, 16)			κ_1 (1,1)	ν (exp(4.85), 1)
6	ω (-2, 16)	$\sigma_2^{(0)}$ (0.8, 0.5)		κ_1 (1,1)	ν (exp(4.85), 1)

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 1190
 1191
 1192
 1193

Table 2: Models, parameters and their prior distributions.

	$\sigma_2^{(0)}$	ω	$\log(\nu)$	$\log(\kappa_1)$
Dataset 1 (baseline, n=80)				
Non-clinical controls: mean(std)	2.5(3.9)	-1.3(2.4)	4.1(1.0)	-0.8(1.4)
Psychotic: mean(std)	3.0(3.9)	-1.4(2.0)	3.1(1.1)	-0.2(0.8)
Clinical controls: mean(std)	1.4(1.9)	-1.2(2.0)	3.3(1.3)	-0.1(1.4)
Kruskal-Wallis Chi Sq (2,80)	2.33, $p=0.31$ $\eta^2=0.02$	0.22, $p=0.9$ $\eta^2=0.0$	11.9, $p=0.003$ $\eta^2=0.15$	9.6, $p=0.008$ $\eta^2=0.12$
Post hoc Dunn tests				

Psychotic vs non-clinical controls	$p(adj)=0.3$	$p(adj)=1$	$p(adj)=0.002$	$p(adj)=0.01$
Clinical vs non-clinical controls	$p(adj)=0.2$	$p(adj)=0.7$	$p(adj)=0.01$	$p(adj)=0.01$
Psychotic vs clinical controls	$p(adj)=0.2$	$p(adj)=0.5$	$p(adj)=0.3$	$p(adj)=0.4$
Dataset 1 (follow-up, n=55)				
Non-clinical controls: mean(std)	2.8(3.4)	-0.9(2.0)	3.6(0.8)	-1.2(1.1)
Psychotic: mean(std)	3.2(3.7)	-1.4(1.5)	2.5(1.2)	-0.3(0.8)
Clinical controls: mean(std)	1.2(0.9)	-1.1(2.0)	3.5(1.1)	-0.5(1.4)
Kruskal-Wallis Chi Sq (2,80)	2.35, $p=0.3$ $\eta^2=0.04$	2.32, $p=0.3$ $\eta^2=0.04$	8.5, $p=0.01$ $\eta^2=0.16$	8.0, $p=0.02$ $\eta^2=0.15$
Post hoc Dunn tests				
Psychotic vs non-clinical controls	$p(adj)=0.4$	$p(adj)=0.2$	$p(adj)=0.01$	$p(adj)=0.007$
Clinical vs non-clinical controls	$p(adj)=0.2$	$p(adj)=0.3$	$p(adj)=0.5$	$p(adj)=0.1$
Psychotic vs clinical controls	$p(adj)=0.3$	$p(adj)=0.3$	$p(adj)=0.01$	$p(adj)=0.1$
Dataset 2 (n=167)				
Non-clinical controls: mean(std)	3.1(2.6)	-2.3(2.0)	2.8(1.0)	-0.8(0.9)
Scz: mean(std)	1.9(1.5)	-2.1(1.8)	2.1(1.2)	0.2(1.0)
Mann-Whitney U test	$Z=3.1$, $p=0.002$, $r=0.24$	$Z=-0.6$, $p=0.6$, $r=0.04$	$Z=3.9$, $p=0.0001$, $r=0.3$	$Z=-5.6$, $p=3 \times 10^{-8}$, $r=0.43$
Dataset 2				

(better-matched controls, n=116)				
Non-clinical controls: mean(std)	2.8(2.7)	-2.2(2.1)	2.9(1.1)	-0.6(1.0)
Scz: mean(std)	1.9(1.5)	-2.1(1.8)	2.1(1.2)	0.2(1.0)
Mann-Whitney U test	Z=1.9, p=0.056, r=0.18	Z=0.12, p=0.9, r=0.01	Z=3.4, p=0.0007, r=0.31	Z=-4.1, p=0.00004, r=0.38

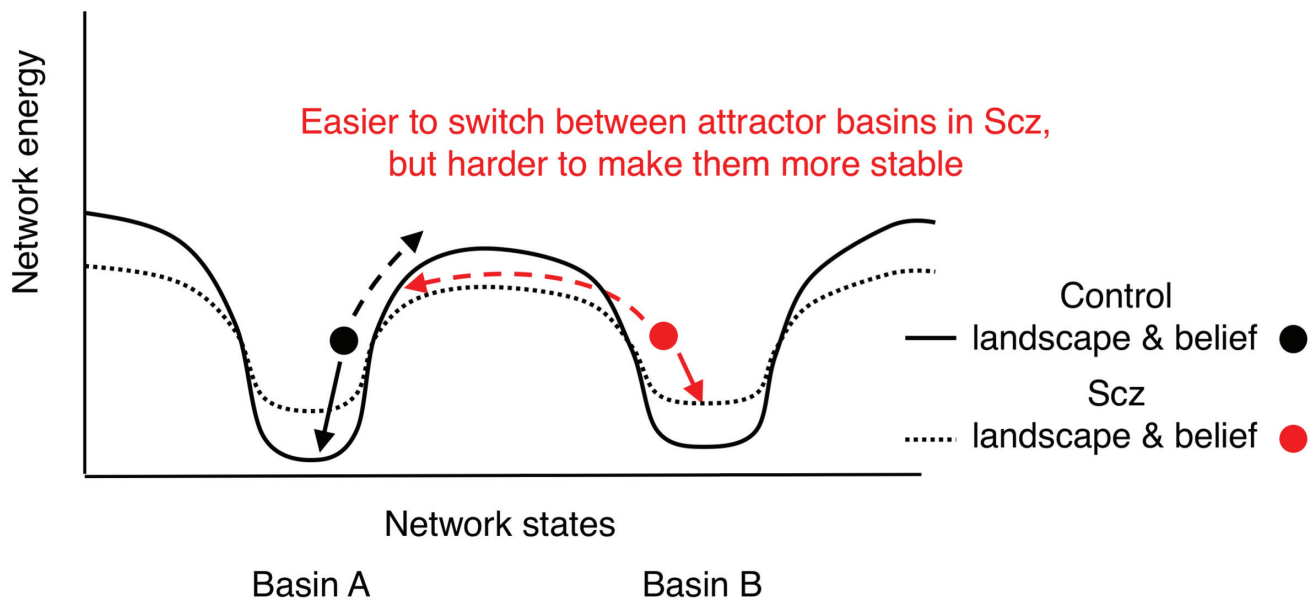
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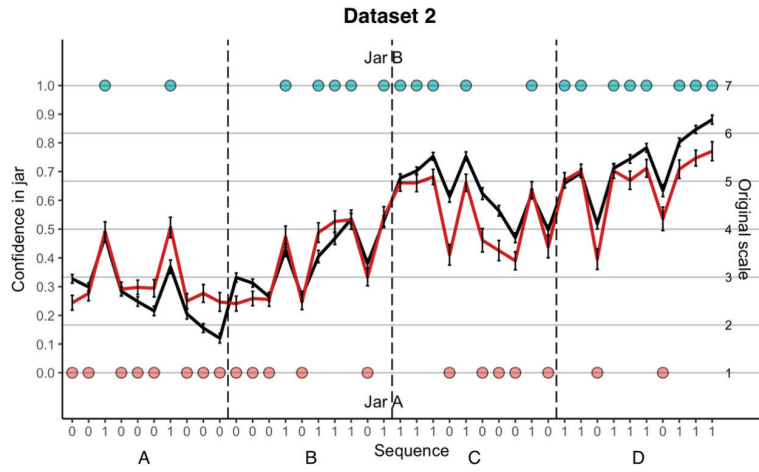
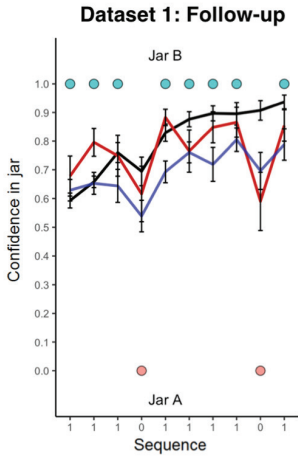
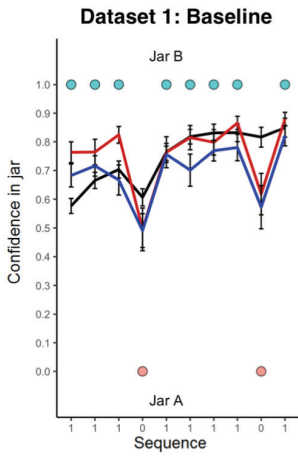
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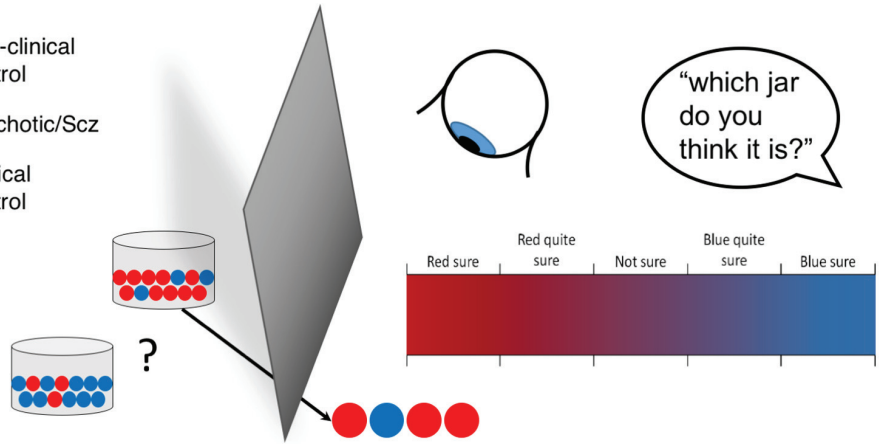
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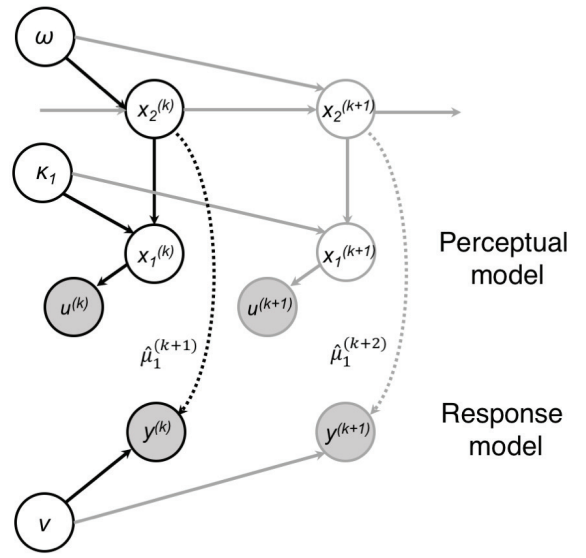
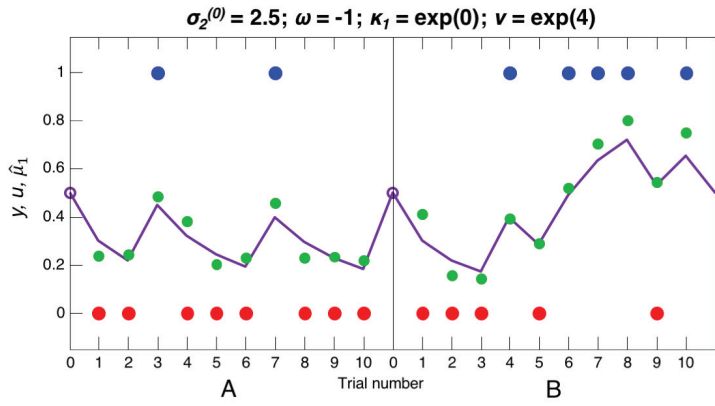
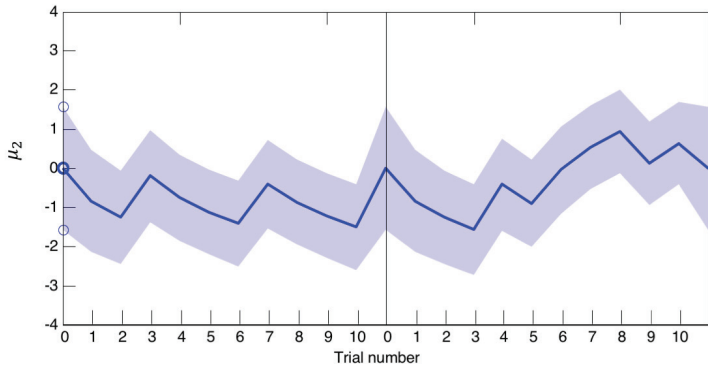
Table 3: Parameter distributions and statistical tests in Datasets 1 and 2

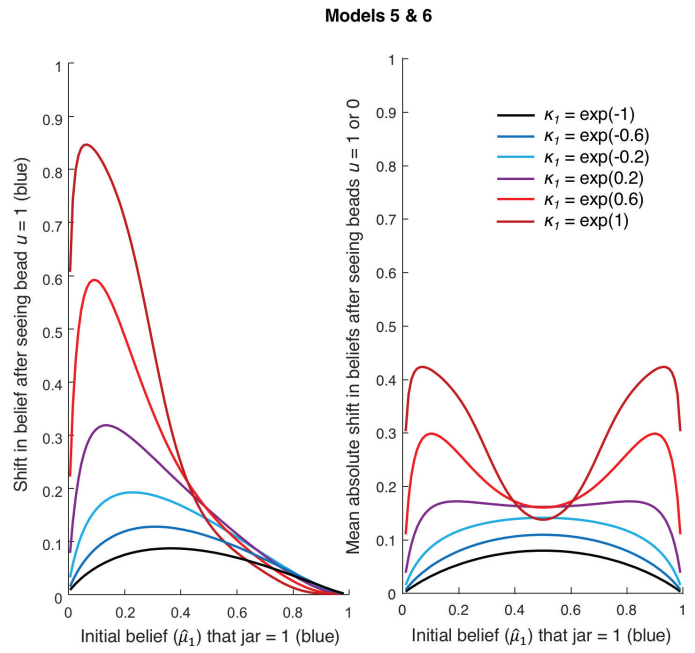
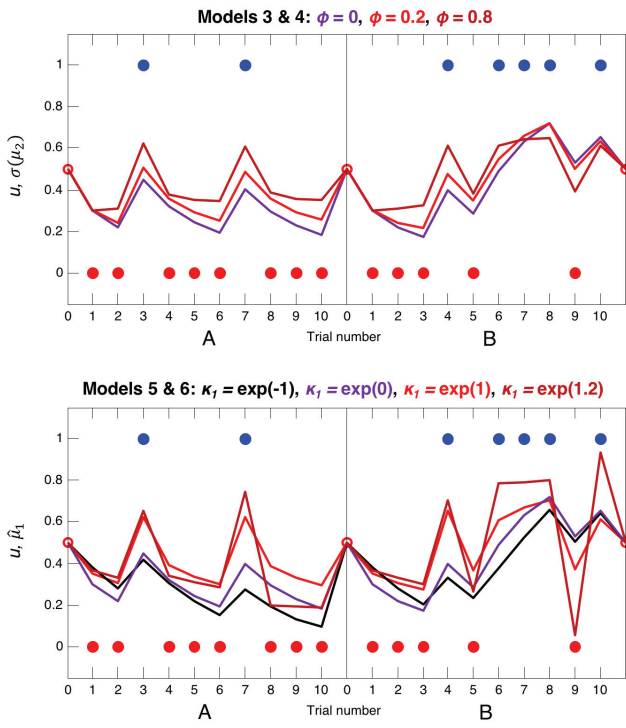


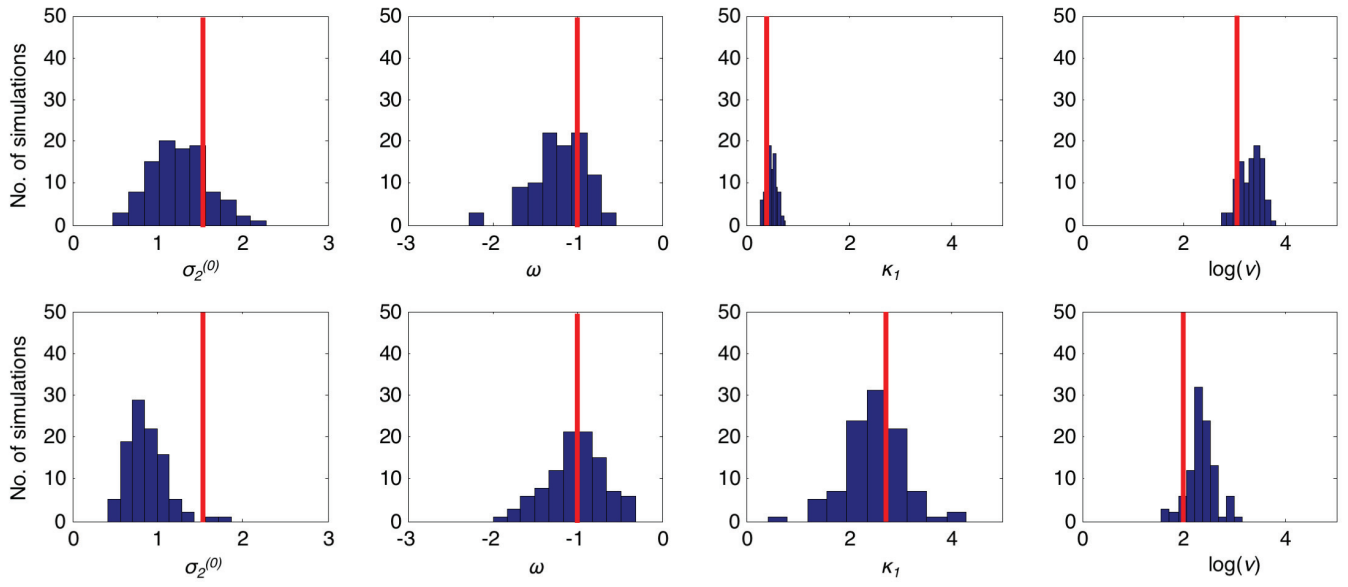


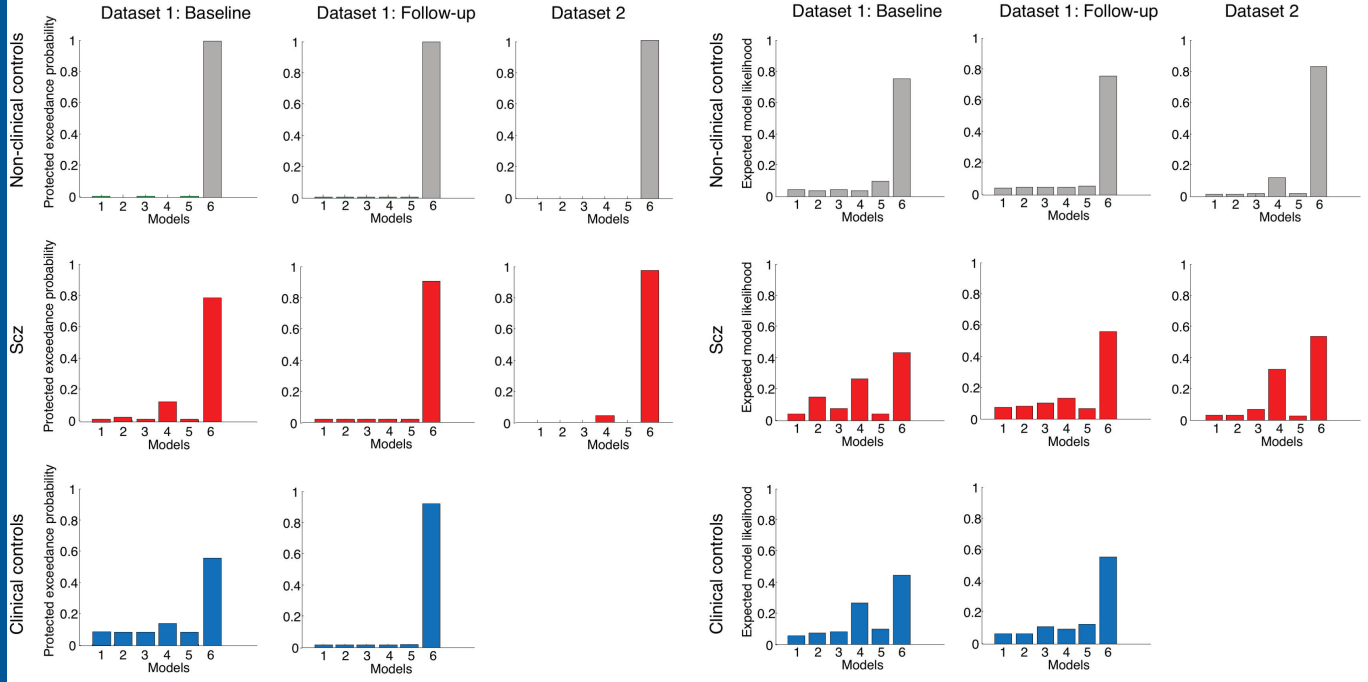
- Non-clinical control
- Psychotic/Scz
- Clinical control

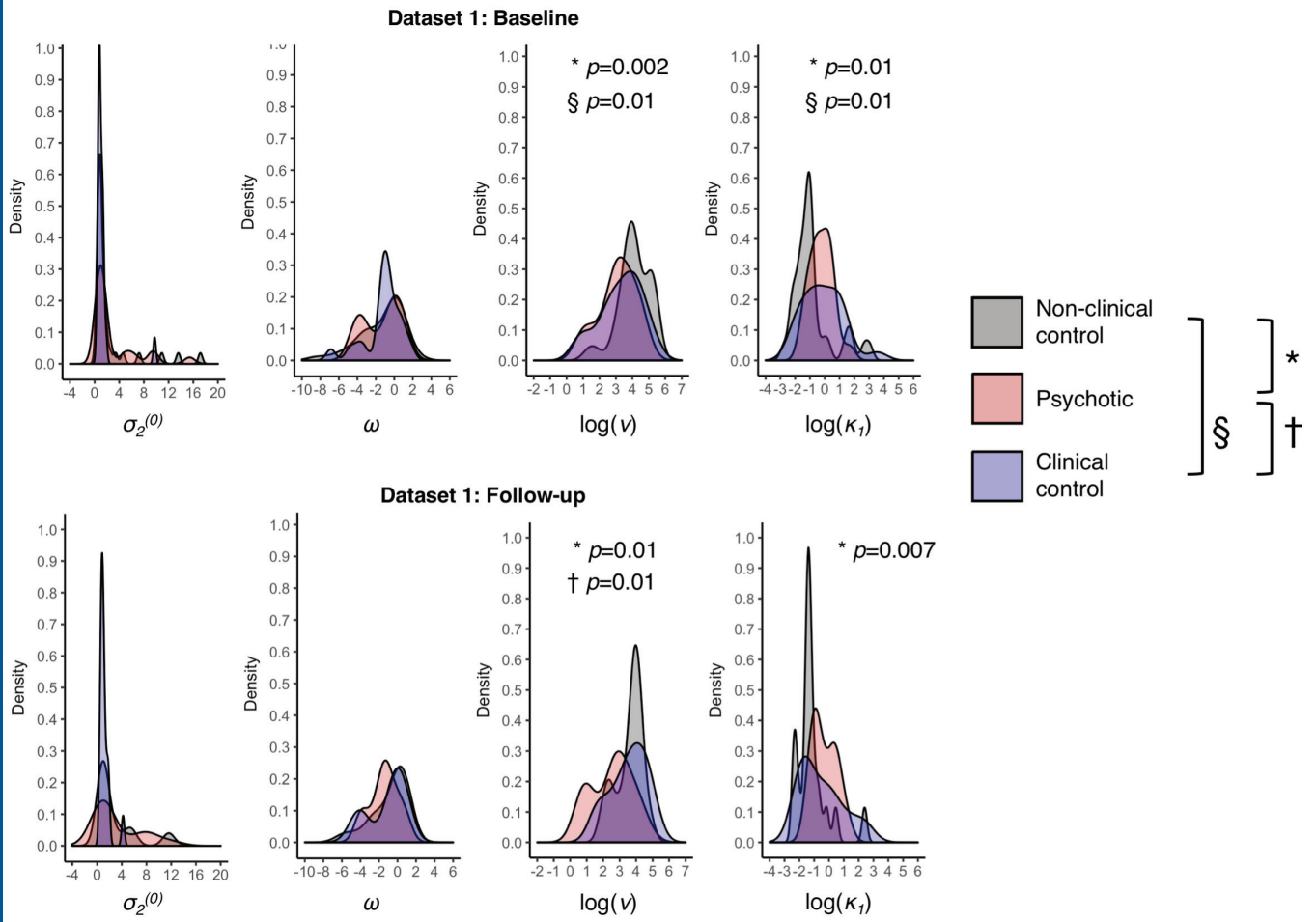




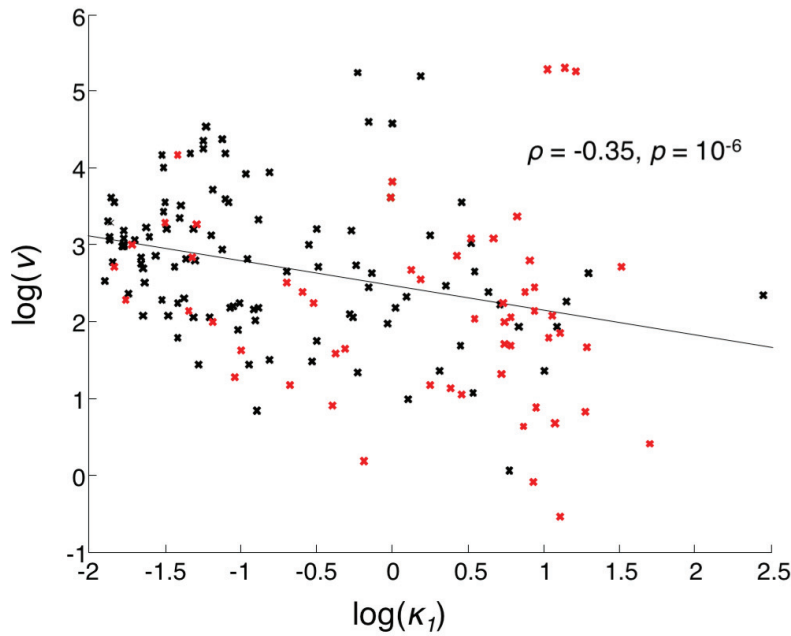
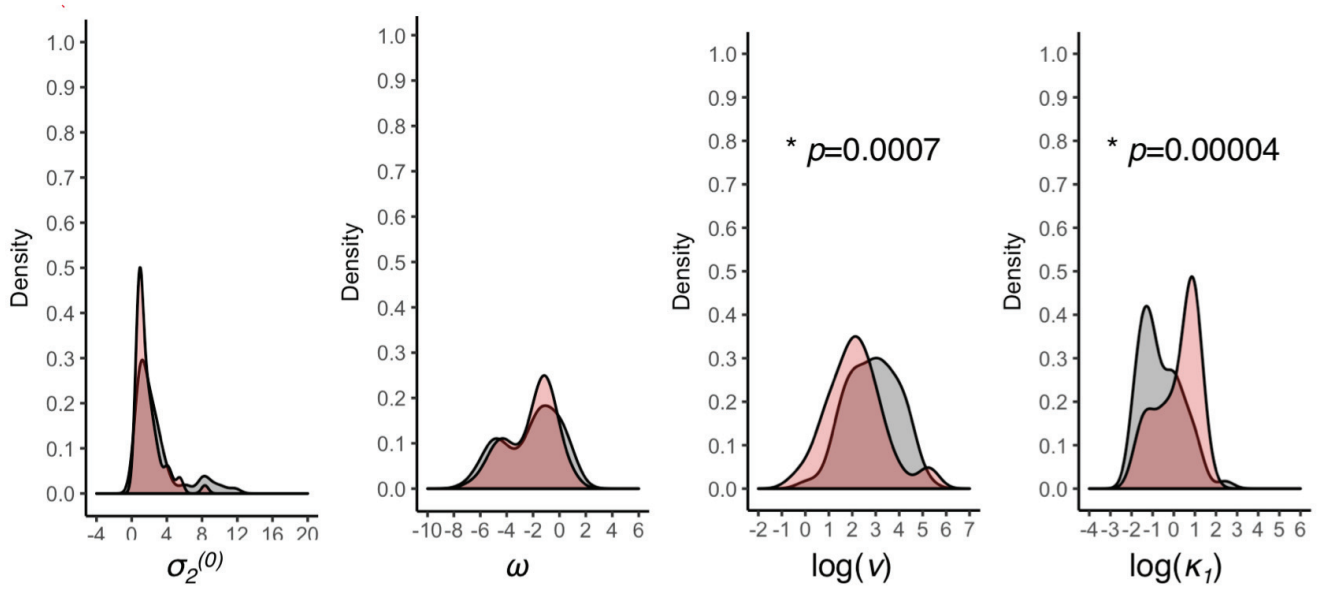




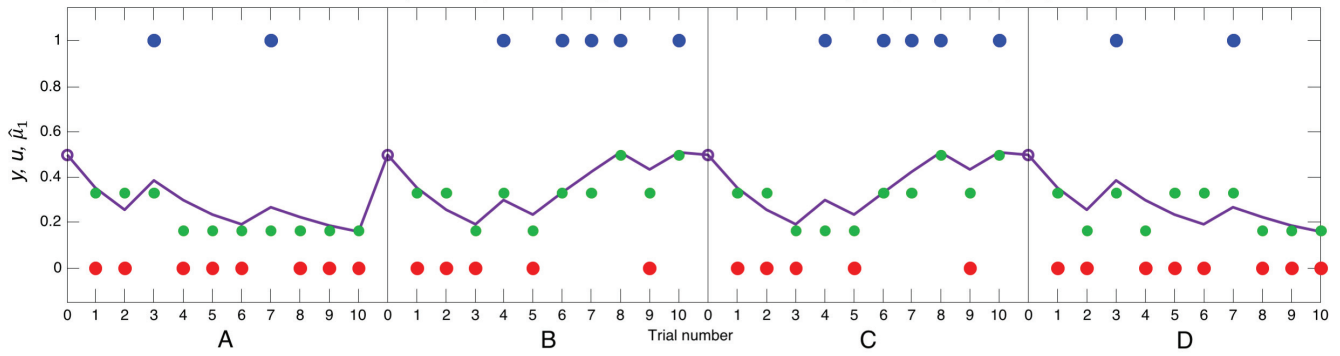




Dataset 2



Subject 18 (control): $\sigma_2^{(0)} = 3.1$; $\omega = -5.2$; $v = \exp(3.9)$; $\kappa_1 = \exp(-0.8)$



Subject 67 (control): $\sigma_2^{(0)} = 4.5$; $\omega = -0.87$; $v = \exp(2.9)$; $\kappa_1 = \exp(-1.1)$

