DCM for fMRI – Advanced topics

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Overview

- DCM: basic concepts
- Evolution of DCM for fMRI
- Bayesian model selection (BMS)
- Translational Neuromodeling

Dynamic causal modeling (DCM)



Generative model



- 1. enforces mechanistic thinking: how could the data have been caused?
- 2. generate synthetic data (observations) by sampling from the prior can model explain certain phenomena at all?
- 3. inference about model structure: formal approach to disambiguating mechanisms $\rightarrow p(m|y)$ or p(y|m)
- 4. inference about parameters $\rightarrow p(\theta|y)$



Stephan et al. 2015, Neuron

Bayesian system identification

Neural dynamics

Observer function

$$dx/dt = f(x, u, \theta)$$

u(t)

$$y = g(x, \theta) + \varepsilon$$

$$\begin{split} p(y \mid \theta, m) &= N(g(\theta), \Sigma(\theta)) \\ p(\theta, m) &= N(\mu_{\theta}, \Sigma_{\theta}) \end{split}$$

Inference on model structure

Inference on parameters

$$p(y \mid m) = \int p(y \mid \theta, m) p(\theta) d\theta$$
$$p(\theta \mid y, m) = \frac{p(y \mid \theta, m) p(\theta, m)}{p(y \mid m)}$$



Variational Bayes (VB)

Idea: find an approximate density $q(\theta)$ that is maximally similar to the true posterior $p(\theta|y)$.

This is often done by assuming a particular form for q (fixed form VB) and then optimizing its sufficient statistics.



Variational Bayes



F(q, y) is a functional wrt. the approximate posterior $q(\theta)$.

Maximizing F(q, y) is equivalent to:

- minimizing KL[q||p]
- tightening F(q, y) as a lower bound to the log model evidence

When F(q, y) is maximized, $q(\theta)$ is our best estimate of the posterior.



Mean field assumption

Factorize the approximate posterior $q(\theta)$ into independent partitions:

$$q(\theta) = \prod_i q_i(\theta_i)$$

where $q_i(\theta_i)$ is the approximate posterior for the *i*th subset of parameters.

For example, split parameters and hyperparameters:

$$p(\theta, \lambda | y) \approx q(\theta, \lambda) = q(\theta)q(\lambda)$$



Jean Daunizeau, www.fil.ion.ucl.ac.uk/ ~jdaunize/presentations/Bayes2.pdf

VB in a nutshell (mean-field approximation)

 Neg. free-energy approx. to model evidence.

$$\ln p(y|m) = F + KL[q(\theta,\lambda), p(\theta,\lambda|y)]$$
$$F = \left\langle \ln p(y,\theta,\lambda) \right\rangle_{q} - KL[q(\theta,\lambda), p(\theta,\lambda|m)]$$

Mean field approx.

$$p(\theta, \lambda | y) \approx q(\theta, \lambda) = q(\theta)q(\lambda)$$

Maximise neg. free energy wrt. q = minimise divergence, by maximising variational energies

$$q(\theta) \propto \exp(I_{\theta}) = \exp\left[\left\langle \ln p(y,\theta,\lambda) \right\rangle_{q(\lambda)}\right]$$
$$q(\lambda) \propto \exp(I_{\lambda}) = \exp\left[\left\langle \ln p(y,\theta,\lambda) \right\rangle_{q(\theta)}\right]$$

 Iterative updating of sufficient statistics of approx. posteriors by gradient ascent.

DCM: methodological developments

- Local extrema → global optimisation schemes for model inversion
 - MCMC
 (Gupta et al. 2015, NeuroImage)
 - Gaussian processes
 (Lomakina et al. 2015, NeuroImage)



- Sampling-based estimates of model evidence
 - Aponte et al. 2015, J. Neurosci. Meth.
 - Raman et al., in preparation
- Choice of priors \rightarrow empirical Bayes
 - Friston et al., submitted
 - Raman et al., submitted



mpdcm: massively parallel DCM



www.translationalneuromodeling.org/tapas

Aponte et al. 2015., J. Neurosci Meth.

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The evolution of DCM in SPM

- DCM is not one specific model, but a framework for Bayesian inversion of dynamic system models
- The implementation in SPM has been evolving over time, e.g.
 - improvements of numerical routines (e.g., optimisation scheme)
 - change in priors to cover new variants (e.g., stochastic DCMs)
 - changes of hemodynamic model



To enable replication of your results, you should ideally state which SPM version (release number) you are using when publishing papers.

Factorial structure of model specification

- Three dimensions of model specification:
 - bilinear vs. nonlinear
 - single-state vs. two-state (per region)
 - deterministic vs. stochastic

A Dynamic Causal Modelling		
Model options:		
modulatory effects	bilinear	nonlinear
Jynamic Causal Modelling	- 🗆 ×	
Model options:		
modulatory effects biline	ear	
states per region	two	
appic Causal Modelling	1	
antions:		
modulatory effects bilinear		
states per region one		
stochastic effects no yes		
	1	

Model





Two-dimensional Taylor series (around $x_0=0$, $u_0=0$):

$$\frac{dx}{dt} = f(x,u) \approx f(x_0,0) + \frac{\partial f}{\partial x}x + \frac{\partial f}{\partial u}u + \frac{\partial^2 f}{\partial x \partial u}ux + \frac{\partial^2 f}{\partial x^2}\frac{x^2}{2} + \dots$$

Bilinear state equation:

$$\frac{dx}{dt} = \left(A + \sum_{i=1}^{m} u_i B^{(i)}\right) x + Cu$$

Nonlinear state equation:

$$\frac{dx}{dt} = \left(A + \sum_{i=1}^{m} u_i B^{(i)} + \sum_{j=1}^{n} x_j D^{(j)}\right) x + Cu$$





Nonlinear Dynamic Causal Model for fMRI

$$\frac{dx}{dt} = \left(A + \sum_{i=1}^{m} u_i B^{(i)} + \sum_{j=1}^{n} x_j D^{(j)}\right) x + Cu$$

Stephan et al. 2008, NeuroImage



Two-state DCM



Stochastic DCM

$$\frac{dx}{dt} = (A + \sum_{j} u_{j} B^{(j)}) x + Cv + \omega^{(x)}$$
$$v = u + \omega^{(v)}$$

- all states are represented in generalised coordinates of motion
- random state fluctuations w^(x) account for endogenous fluctuations, have unknown precision and smoothness → two hyperparameters
- fluctuations w^(v) induce uncertainty about how inputs influence neuronal activity
- · can be fitted to resting state data

Estimates of hidden causes and states (Generalised filtering)



Li et al. 2011, NeuroImage

Spectral DCM

- <u>deterministic</u> model that generates predicted cross-spectra in a distributed neuronal network or graph
- finds the effective connectivity among hidden neuronal states that best explains the observed functional connectivity among hemodynamic responses
- advantage:
 - replaces an optimisation problem wrt. stochastic differential equations with a deterministic approach from linear systems theory
 → computationally very efficient
- disadvantages:
 - assumes stationarity

Cross-correlation & convolution

- cross-correlation = measure of similarity of two waveforms as a function of the time-lag of one relative to the other
 - slide two functions over each other and measure overlaps at all lags
- related to the pdf of the difference beteween two random variables
 → a general measure of similarity between two time series

$$(f \star g)(\tau) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f^*(t) g(t+\tau) dt$$



Source: Wikipedia

cross-spectra

= Fourier transform of cross-correlation

cross-correlation

= generalized form of correlation (at zero lag, this is the conventional measure of functional connectivity)



Friston et al. 2014, NeuroImage

"All models are wrong, but some are useful."



George E.P. Box (1919-2013)

Hierarchical strategy for model validation



For DCM: >15 published validation studies (incl. 6 animal studies):

- infers site of seizure origin (David et al. 2008)
- infers primary recipient of vagal nerve stimulation (Reyt et al. 2010)
- infers synaptic changes as predicted by microdialysis (Moran et al. 2008)
- infers fear conditioning induced plasticity in amygdala (Moran et al. 2009)
- tracks anaesthesia levels (Moran et al. 2011)
- predicts sensory stimulation (Brodersen et al. 2010)

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Generative models & model selection

- any DCM = a particular generative model of how the data (may) have been caused
- generative modelling: comparing competing hypotheses about the mechanisms underlying observed data
 - \rightarrow a priori definition of hypothesis set (model space) is crucial
 - → determine the most plausible hypothesis (model), given the data
- model selection ≠ model validation!
 - → model validation requires external criteria (external to the measured data)

Model comparison and selection

Given competing hypotheses on structure & functional mechanisms of a system, which model is the best?

Which model represents the best balance between model fit and model complexity?

For which model m does p(y|m) become maximal?



Pitt & Miyung (2002) TICS

Bayesian model selection (BMS)

Model evidence (marginal likelihood):

$$p(y \mid m) = \int p(y \mid \theta, m) p(\theta \mid m) \ d\theta$$



accounts for both accuracy and complexity of the model

 "If I randomly sampled from my prior and plugged the resulting value into the likelihood function, how close would the predicted data be – on average – to my observed data?"



Various approximations, e.g.:

- negative free energy, AIC, BIC

McKay 1992, *Neural Comput.* Penny et al. 2004a, *NeuroImage*

Model space (hypothesis set) *M*

Model space M is defined by prior on models. Usual choice: flat prior over a small set of models.

$$p(m) = \begin{cases} 1/|M| \text{ if } m \in M \\ 0 \text{ if } m \notin M \end{cases}$$

In this case, the posterior probability of model i is:

$$p(m_i \mid y) = \frac{p(y \mid m_i) p(m_i)}{\sum_{j=1}^{|M|} p(y \mid m_j) p(m_j)} = \frac{p(y \mid m_i)}{\sum_{j=1}^{|M|} p(y \mid m_j)}$$

Approximations to the model evidence in DCM

Logarithm is a monotonic function



Maximizing log model evidence = Maximizing model evidence

Log model evidence = balance between fit and complexity

$$\log p(y | m) = accuracy(m) - complexity(m)$$

= log p(y | \theta, m) - complexity(m)
Akaike Information Criterion: AIC = log p(y | \theta, m) - p
Bayesian Information Criterion: BIC = log p(y | \theta, m) - \frac{p}{2} log N

The (negative) free energy approximation ${\it F}$

F is a lower bound on the log model evidence, where the bound is determined by the KL divergence between an approximate posterior q and the true posterior::

$$\log p(y | m) = F + KL[q(\theta), p(\theta | y, m)]$$

Like AIC/BIC, F is an accuracy/complexity tradeoff:

$$F = \underbrace{\langle \log p(y | \theta, m) \rangle}_{accuracy} - \underbrace{KL[q(\theta), p(\theta | m)]}_{complexity}$$



The complexity term in F

• In contrast to AIC & BIC, the complexity term of the negative free energy *F* accounts for parameter interdependencies.

$$KL[q(\theta), p(\theta \mid m)] = \frac{1}{2} \ln |C_{\theta|y}| + \frac{1}{2} (\mu_{\theta|y} - \mu_{\theta})^T C_{\theta}^{-1} (\mu_{\theta|y} - \mu_{\theta})$$

- determinant = measure of "volume" (space spanned by the eigenvectors of the matrix)
- The complexity term of *F* is higher
 - the more independent the prior parameters (\uparrow effective DFs)
 - the more dependent the posterior parameters
 - the more the posterior mean deviates from the prior mean

Bayes factors

To compare two models, we could just compare their log evidences.

But: the log evidence is just some number – not very intuitive!

A more intuitive interpretation of model comparisons is made possible by Bayes factors:

positive value, [0; ∞ [

$$B_{12} = \frac{p(y \mid m_1)}{p(y \mid m_2)}$$

Kass & Raftery classification:

B ₁₂	p(m₁ y)	Evidence
1 to 3	50-75%	weak
3 to 20	75-95%	positive
20 to 150	95-99%	strong
≥ 150	≥ 99%	Very strong

Kass & Raftery 1995, J. Am. Stat. Assoc.

Fixed effects BMS at group level

Group Bayes factor (GBF) for 1...K subjects:

$$GBF_{ij} = \prod_{k} BF_{ij}^{(k)}$$

Average Bayes factor (ABF):

$$ABF_{ij} = \sqrt[K]{\prod_{k} BF_{ij}^{(k)}}$$

Problems:

- blind with regard to group heterogeneity
- sensitive to outliers

Random effects BMS for heterogeneous groups


Random effects BMS for heterogeneous groups



Stephan et al. 2009a, *NeuroImage* Penny et al. 2010, *PLoS Comp. Biol.*



Log model evidence differences

Subjects



Stephan et al. 2009a, NeuroImage

How can we report the results of random effects BMS?

1. Dirichlet parameter estimates

2. expected posterior probability of obtaining the k-th model for any randomly selected subject

$$\langle r_k \rangle_q = \alpha_k / (\alpha_1 + \ldots + \alpha_K)$$

 $\boldsymbol{\alpha}$

- 3. **exceedance probability** that a particular model *k* is more likely than any other model (of the *K* models tested), given the group data
- 4. protected exceedance probability: see below

$$\exists k \in \{1...K\}, \forall j \in \{1...K \mid j \neq k\}:$$
$$\varphi_k = p(r_k > r_j \mid y; \alpha)$$

Overfitting at the level of models

- \uparrow #models \Rightarrow \uparrow risk of overfitting
- solutions:
 - regularisation: definition of model space = choosing priors p(m)
 - family-level BMS
 - Bayesian model averaging (BMA)



posterior model probability:

$$p(m \mid y) = \frac{p(y \mid m) p(m)}{\sum_{m} p(y \mid m) p(m)}$$

BMA:

$$p(\theta | y)$$

 $= \sum_{m} p(\theta | y, m) p(m | y)$

Model space partitioning or: Comparing model families

- partitioning model space into K subsets or families:
- pooling information over all models in these subsets allows one to compute the probability of a model family, given the data
- effectively removes uncertainty about any aspect of model structure, other than the attribute of interest (which defines the partition)

$$M = \left\{ f_1, \dots, f_K \right\}$$

 $p(f_k)$

Stephan et al. 2009, *NeuroImage* Penny et al. 2010, *PLoS Comput. Biol.*

Family-level inference: fixed effects

- We wish to have a uniform prior at the family level:
- This is related to the model level via the sum of the priors on models:
- Hence the uniform prior at the family level is:
- The probability of each family is then obtained by summing the posterior probabilities of the models it includes:

$$p(f_k) = \frac{1}{K}$$

$$p(f_k) = \sum_{m \in f_k} p(m)$$

$$\forall m \in f_k : p(m) = \frac{1}{K |f_k|}$$

$$p(f_k | y_{1..N}) = \sum_{m \in f_k} p(m | y_{1..N})$$

Family-level inference: random effects

- The frequency of a family in the population is given by:
- In RFX-BMS, this follows a Dirichlet distribution, with a uniform prior on the parameters α (see above).
- A uniform prior over family probabilities can be obtained by setting:

$$s_k = \sum_{m \in f_k} r_m$$

$$p(s) = Dir(\alpha)$$

$$\forall m \in f_k : \alpha_{prior}(m) = \frac{1}{|f_k|}$$

Stephan et al. 2009, *NeuroImage* Penny et al. 2010, *PLoS Comput. Biol.*

Family-level inference: random effects – a special case

• When the families are of equal size, one can simply sum the posterior model probabilities within families by exploiting the agglomerative property of the Dirichlet distribution:

$$(r_1, r_2, ..., r_K) \sim Dir(\alpha_1, \alpha_2, ..., \alpha_K)$$

$$\Rightarrow r_1^* = \sum_{k \in N_1} r_k, r_2^* = \sum_{k \in N_2} r_k, ..., r_J^* = \sum_{k \in N_J} r_k$$

$$\sim Dir\left(\alpha_1^* = \sum_{k \in N_1} \alpha_k, \alpha_2^* = \sum_{k \in N_2} \alpha_k, ..., \alpha_J^* = \sum_{k \in N_J} \alpha_k\right)$$



nonlinear models linear models

Bayesian Model Averaging (BMA)

- abandons dependence of parameter inference on a single model and takes into account model uncertainty
- represents a particularly useful alternative
 - when none of the models (or model subspaces) considered clearly outperforms all others
 - when comparing groups for which the optimal model differs

single-subject BMA:

$$p(\theta \mid y) = \sum_{m} p(\theta \mid y, m) p(m \mid y)$$

group-level BMA:

$$p(\theta_n \mid y_{1..N}) = \sum p(\theta_n \mid y_n, m) p(m \mid y_{1..N})$$

т

NB: $p(m|y_{1..N})$ can be obtained by either FFX or RFX BMS

Protected exceedance probability: Using BMA to protect against chance findings

- EPs express our confidence that the posterior probabilities of models are different under the hypothesis H_1 that models differ in probability: $r_k \neq 1/K$
- does not account for possibility "null hypothesis" H_0 : $r_k=1/K$
- **Bayesian omnibus risk (BOR)** of wrongly accepting H₁ over H₀:

$$P_{o} = \frac{1}{1 + \frac{p(m|H_{1})}{p(m|H_{0}).}}$$

• **protected EP**: Bayesian model averaging over H_0 and H_1 :

$$\begin{split} \widetilde{\varphi}_{k} &= P(r_{k} \geq r_{k' \neq k} | y) \\ &= P(r_{k} \geq r_{k' \neq k} | y, H_{1}) P(H_{1} | y) + P(r_{k} \geq r_{k' \neq k} | y, H_{0}) P(H_{0} | y) \\ &= \varphi_{k}(1 - P_{0}) + \frac{1}{K} P_{0} \end{split}$$

Rigoux et al. 2014, NeuroImage



Stephan et al. 2010, NeuroImage

Two empirical example applications



Michael Breakspear,^{1,2,3,*} Gloria Roberts,^{3,4,*} Melissa J. Green,^{3,4,5,6} Vinh T. Nguyen,¹ Andrew Frankland,^{3,4} Florence Levy,³ Rhoshel Lenroot^{3,6} and Philip B. Mitchell^{3,4}

Original Investigation

Brain Connectivity Abnormalities Predating the Onset of Psychosis Correlation With the Effect of Medication

André Schmidt, PhD; Renata Smieskova, PhD; Jacqueline Aston, MD; Andor Simon, MD; Paul Allen, PhD; Paolo Fusar-Poli, MD, PhD; Philip K. McGuire, MD, PhD; Anita Riecher-Rössler, MD, PhD; Klaas E. Stephan, MD, PhD; Stefan Borgwardt, MD, PhD Breakspear et al. 2015, *Brain*

Schmidt et al. 2013, *JAMA Psychiatry*

Go/No-Go task to emotional faces (bipolar patients, at-risk individuals, controls)

С

- Hypoactivation of left ۲ IFG in the at-risk group during fearful distractor trials
- DCM used to explain ٠ interaction of motor inhibition and fear perception
- That is: what is the ٠ most likely circuit mechanism explaining the fear x inhibition interaction in IFG?



Model space



A: Bilinear models

 models of serial (1-3), parallel (4) and hierarchical (5-8) processes

Family-level BMS

- family-level comparison: nonlinear models more likely than bilinear ones in both healthy controls and bipolar patients
- at-risk group: bilinear models more likely
- significant group difference in ACC modulation of DLPFC→IFG interaction





- 17 at-risk mental state (ARMS) individuals
- 21 first-episode patients (13 non-treated)
- 20 controls

Prefrontal-parietal connectivity during working memory in schizophrenia



Driving input
 Driving input
 Endogenous connection
 Modulatory input

Schmidt et al. 2013, JAMA Psychiatry

BMS results for all groups



Schmidt et al. 2013, JAMA Psychiatry

BMA results: $PFC \rightarrow PPC$ connectivity





17 ARMS, 21 first-episode (13 non-treated), 20 controls

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disease mechanism B

disease mechanism C

Application to brain activity and behaviour of individual patients

Stephan et al. 2015, Neuron

Model-based predictions for single patients

model structure







parameter estimates model-based decoding (generative embedding)



Synaesthesia

- "projectors" experience color externally colocalized with a presented grapheme
- "associators" report an internally evoked association
- across all subjects: no evidence for either model
- but BMS results map precisely onto projectors (bottom-up mechanisms) and associators (top-down)



Generative embedding (supervised): classification



Brodersen et al. 2011, PLoS Comput. Biol.

Discovering remote or "hidden" brain lesions



Discovering remote or "hidden" brain lesions



Connectional fingerprints : aphasic patients (N=11) vs. controls (N=26)

6-region DCM of auditory areas during passive speech listening





Brodersen et al. 2011, PLoS Comput. Biol.

Can we predict presence/absence of the "hidden" lesion?







Repeat *n* times:

1 ROI definition and *n* **model inversions** unbiased estimate

Generative embedding (unsupervised): detecting patient subgroups





- 42 controls vs. 41 schizophrenic patients
- fMRI data from working memory task (Deserno et al. 2012, J. Neurosci)

Detecting subgroups of patients in schizophrenia

- three distinct subgroups (total N=41)
- subgroups differ (p < 0.05) wrt. negative symptoms on the positive and negative symptom scale (PANSS)









Further reading: DCM for fMRI and BMS – part 1

- Aponte EA, Raman S, Sengupta B, Penny WD, Stephan KE, Heinzle J (2015) mpdcm: A Toolbox for Massively Parallel Dynamic Causal Modeling. Journal of Neuroscience Methods, in press
- Breakspear M, Roberts G, Green MJ, Nguyen VT, Frankland A, Levy F, Lenroot R, Mitchell PB (2015) Network dysfunction of emotional and cognitive processes in those at genetic risk of bipolar disorder. Brain, in press.
- Brodersen KH, Schofield TM, Leff AP, Ong CS, Lomakina EI, Buhmann JM, Stephan KE (2011) Generative embedding for model-based classification of fMRI data. PLoS Computational Biology 7: e1002079.
- Brodersen KH, Deserno L, Schlagenhauf F, Lin Z, Penny WD, Buhmann JM, Stephan KE (2014) Dissecting psychiatric spectrum disorders by generative embedding. NeuroImage: Clinical 4: 98-111
- Daunizeau J, David, O, Stephan KE (2011) Dynamic Causal Modelling: A critical review of the biophysical and statistical foundations. NeuroImage 58: 312-322.
- Daunizeau J, Stephan KE, Friston KJ (2012) Stochastic Dynamic Causal Modelling of fMRI data: Should we care about neural noise? NeuroImage 62: 464-481.
- Friston KJ, Harrison L, Penny W (2003) Dynamic causal modelling. NeuroImage 19:1273-1302.
- Friston K, Stephan KE, Li B, Daunizeau J (2010) Generalised filtering. Mathematical Problems in Engineering 2010: 621670.
- Friston KJ, Li B, Daunizeau J, Stephan KE (2011) Network discovery with DCM. NeuroImage 56: 1202–1221.
- Friston K, Penny W (2011) Post hoc Bayesian model selection. Neuroimage 56: 2089-2099.
- Kiebel SJ, Kloppel S, Weiskopf N, Friston KJ (2007) Dynamic causal modeling: a generative model of slice timing in fMRI. NeuroImage 34:1487-1496.
- Li B, Daunizeau J, Stephan KE, Penny WD, Friston KJ (2011) Stochastic DCM and generalised filtering. NeuroImage 58: 442-457
- Lomakina EI, Paliwal S, Diaconescu AO, Brodersen KH, Aponte EA, Buhmann JM, Stephan KE (2015) Inversion of Hierarchical Bayesian models using Gaussian processes. NeuroImage 118: 133-145.
- Marreiros AC, Kiebel SJ, Friston KJ (2008) Dynamic causal modelling for fMRI: a two-state model. NeuroImage 39:269-278.
- Penny WD, Stephan KE, Mechelli A, Friston KJ (2004a) Comparing dynamic causal models. NeuroImage 22:1157-1172.
- Penny WD, Stephan KE, Mechelli A, Friston KJ (2004b) Modelling functional integration: a comparison of structural equation and dynamic causal models. NeuroImage 23 Suppl 1:S264-274.
Further reading: DCM for fMRI and BMS – part 2

- Penny WD, Stephan KE, Daunizeau J, Joao M, Friston K, Schofield T, Leff AP (2010) Comparing Families of Dynamic Causal Models. PLoS Computational Biology 6: e1000709.
- Penny WD (2012) Comparing dynamic causal models using AIC, BIC and free energy. Neuroimage 59: 319-330.
- Rigoux L, Stephan KE, Friston KJ, Daunizeau J (2014) Bayesian model selection for group studies revisited. NeuroImage 84: 971-985.
- Stephan KE, Harrison LM, Penny WD, Friston KJ (2004) Biophysical models of fMRI responses. Curr Opin Neurobiol 14:629-635.
- Stephan KE, Weiskopf N, Drysdale PM, Robinson PA, Friston KJ (2007) Comparing hemodynamic models with DCM. NeuroImage 38:387-401.
- Stephan KE, Harrison LM, Kiebel SJ, David O, Penny WD, Friston KJ (2007) Dynamic causal models of neural system dynamics: current state and future extensions. J Biosci 32:129-144.
- Stephan KE, Weiskopf N, Drysdale PM, Robinson PA, Friston KJ (2007) Comparing hemodynamic models with DCM. NeuroImage 38:387-401.
- Stephan KE, Kasper L, Harrison LM, Daunizeau J, den Ouden HE, Breakspear M, Friston KJ (2008) Nonlinear dynamic causal models for fMRI. NeuroImage 42:649-662.
- Stephan KE, Penny WD, Daunizeau J, Moran RJ, Friston KJ (2009a) Bayesian model selection for group studies. NeuroImage 46:1004-1017.
- Stephan KE, Tittgemeyer M, Knösche TR, Moran RJ, Friston KJ (2009b) Tractography-based priors for dynamic causal models. NeuroImage 47: 1628-1638.
- Stephan KE, Penny WD, Moran RJ, den Ouden HEM, Daunizeau J, Friston KJ (2010) Ten simple rules for Dynamic Causal Modelling. NeuroImage 49: 3099-3109.
- Stephan KE, Iglesias S, Heinzle J, Diaconescu AO (2015) Translational Perspectives for Computational Neuroimaging. Neuron 87: 716-732.

Thank you