

# Bayesian model selection and averaging

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*Methods and Models for fMRI analysis, Practical Session  
Tuesday, November 26<sup>th</sup> 2019*

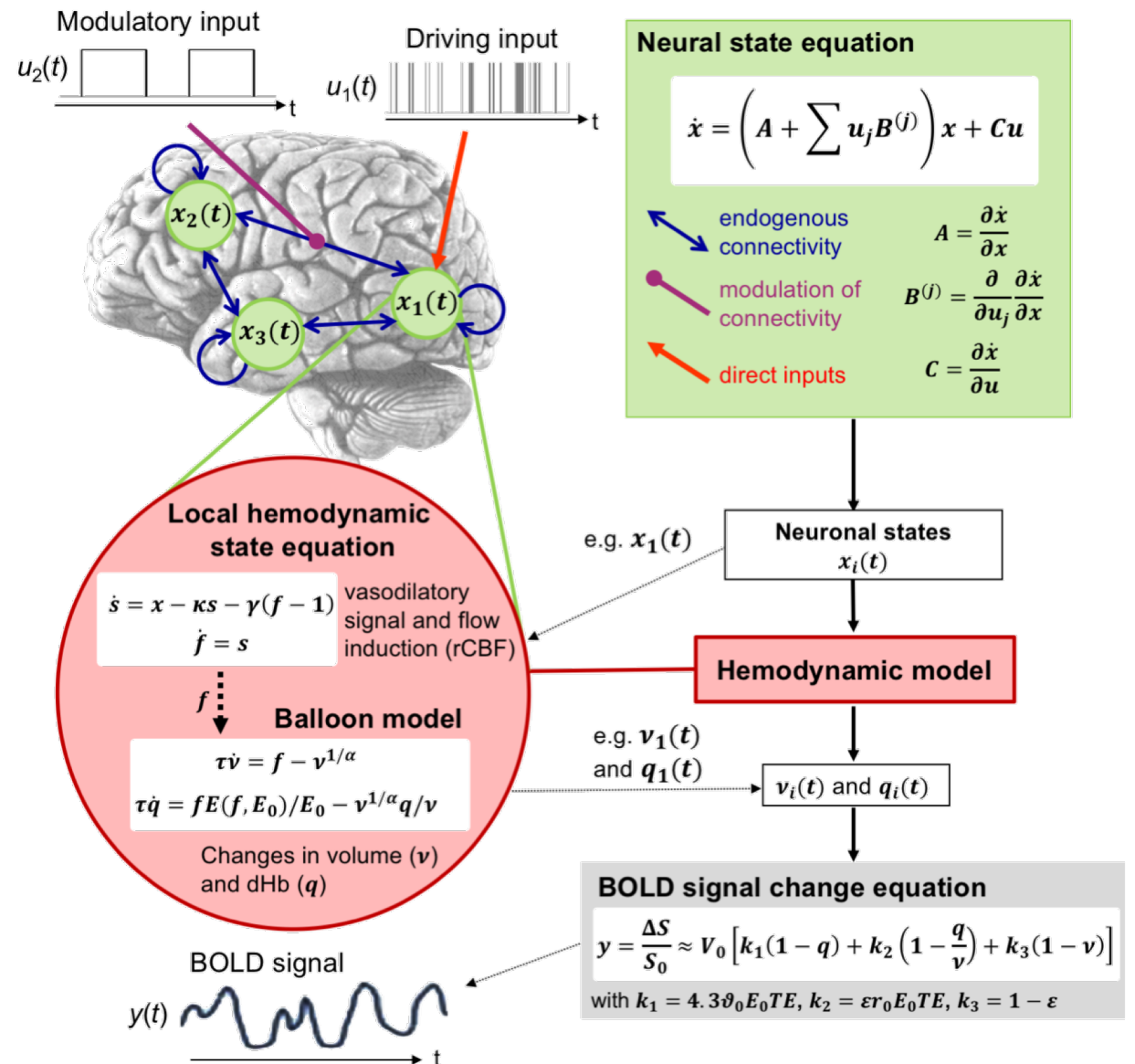
# TODAY'S TUTORIAL

Bayesian model selection and averaging:

- Brief teaser on Dynamic Causal Modeling (DCM)
- Fixed- and random-effects Bayesian model selection (BMS)
- Bayesian model averaging (BMA)

# TEASER: DCM FOR FMRI

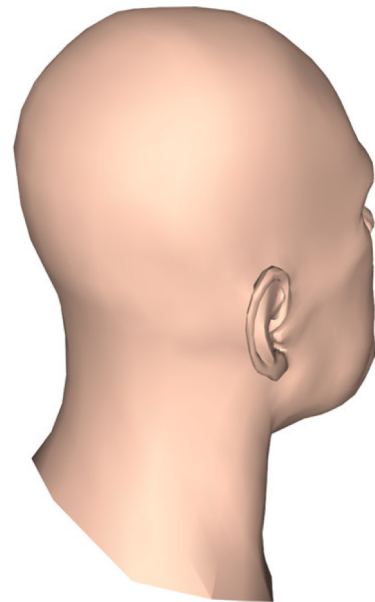
FYI: a more detailed introduction to DCM will follow in two weeks



Friston et al., 2003, *NeuroImage*; Stephan et al., 2015, *Neuron*

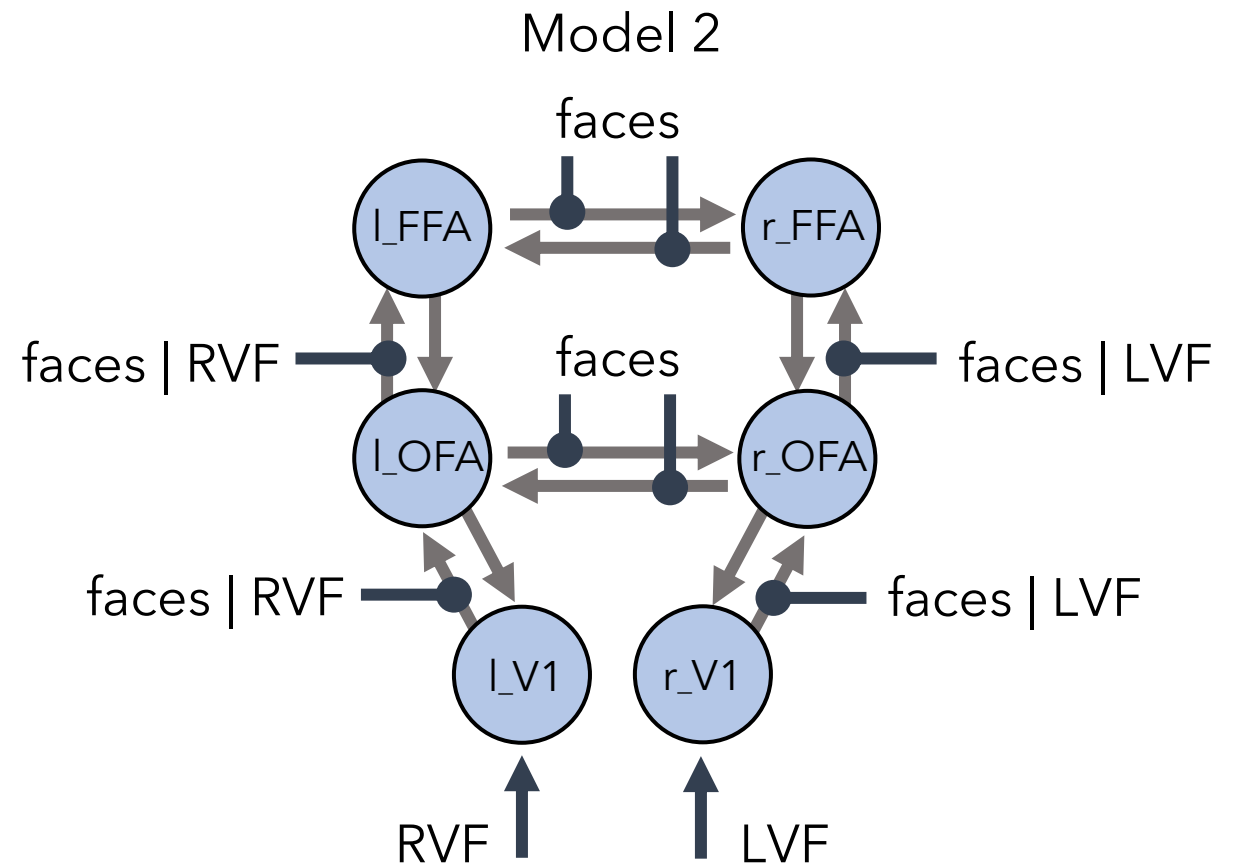
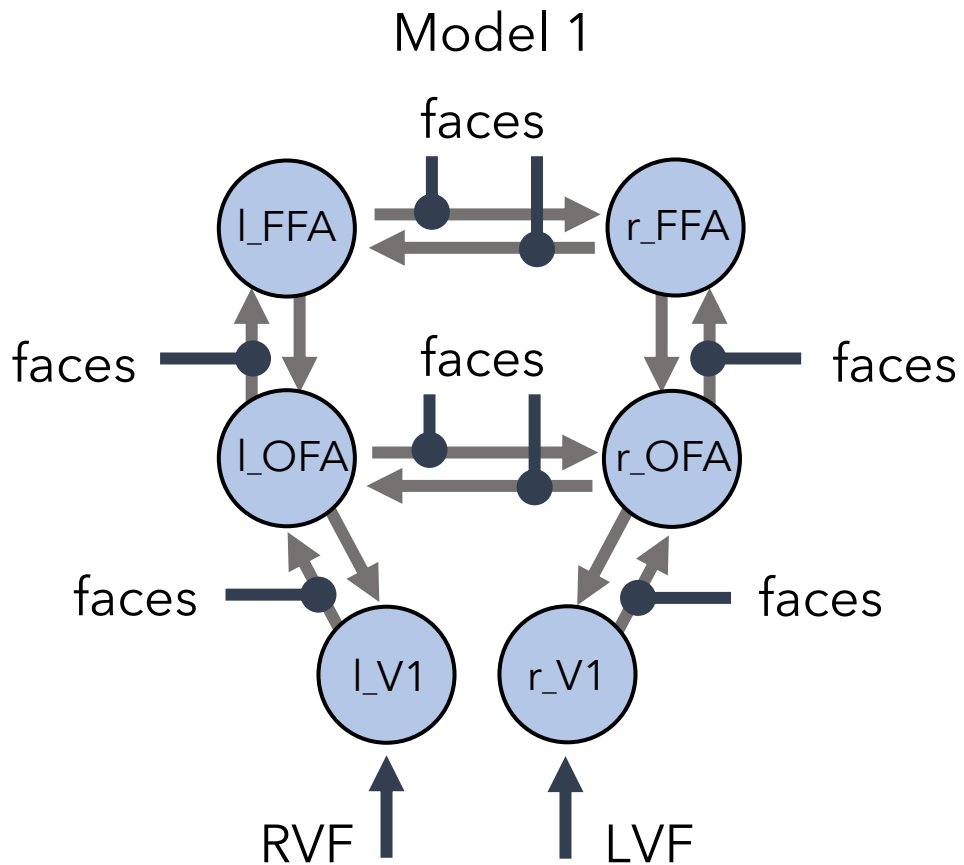
# DYNAMIC CAUSAL MODELING

Different mechanisms in the face perception network:



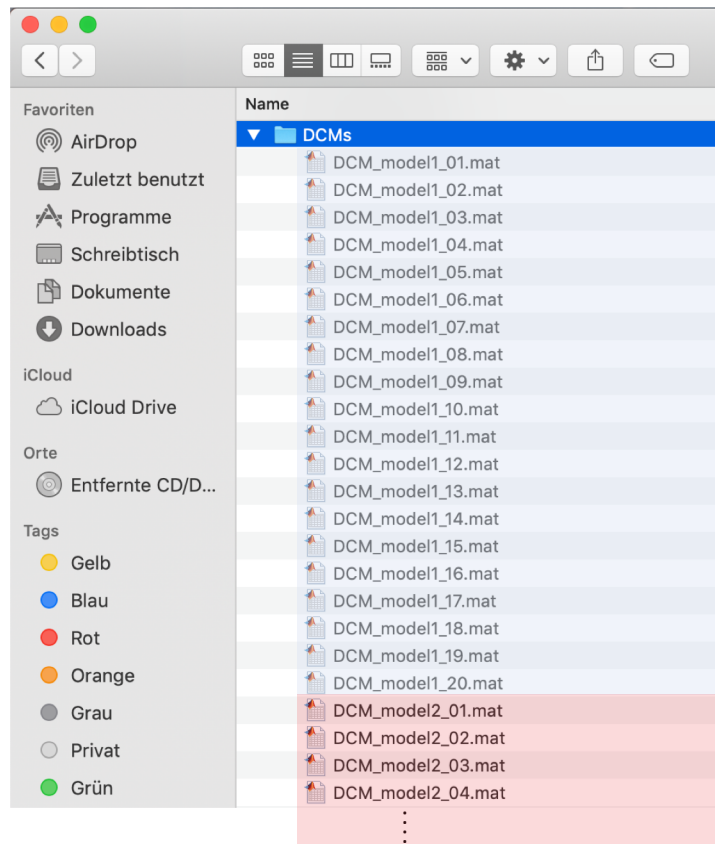
# DYNAMIC CAUSAL MODELING

Different mechanisms in the face perception network:



# ORGANIZATION OF DATA

Data is stored in “**DCM\_model\*.mat**” files:

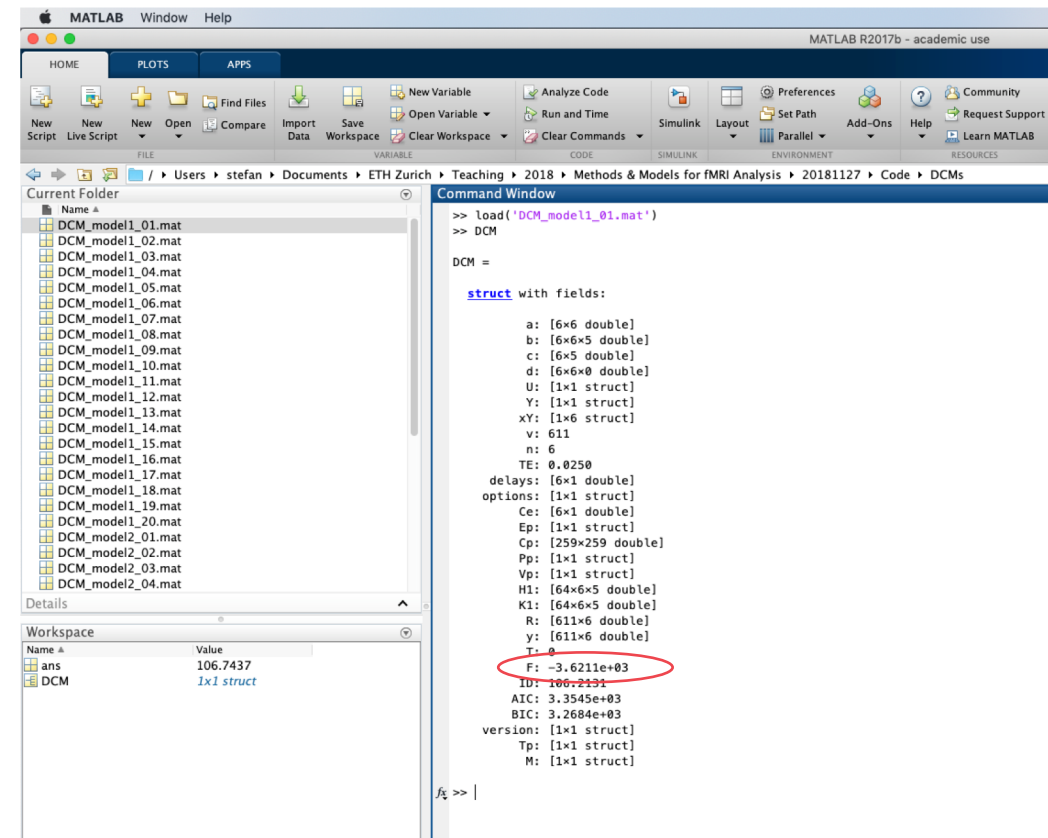
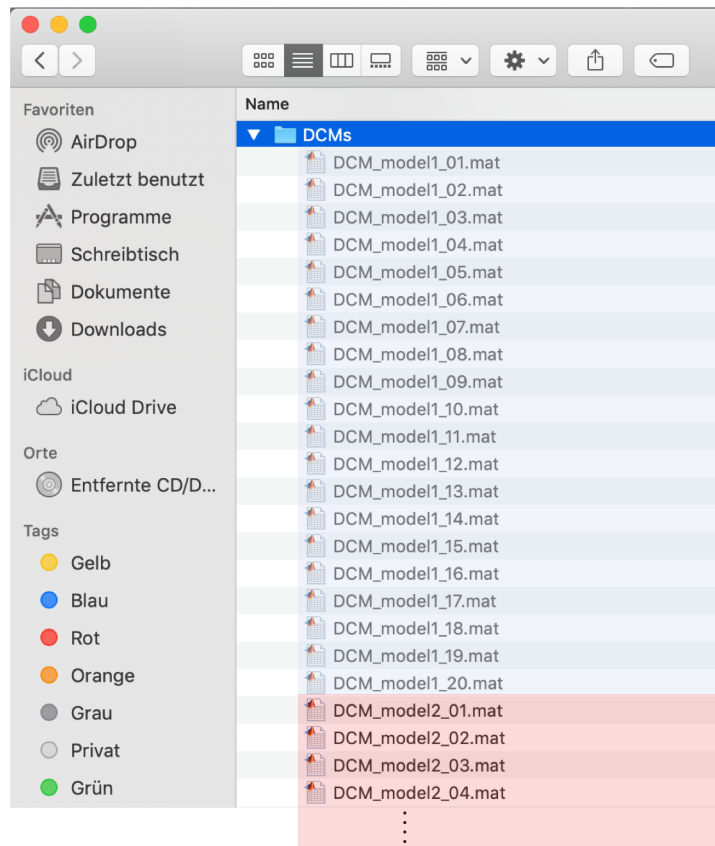


Model 1 for 20 subjects

Model 2 for 20 subjects

# ORGANIZATION OF DATA

Data is stored in “**DCM\_model\*.mat**” files:



# TRY TO SET UP A FIXED-EFFECTS BMS ANALYSIS

Reminder: **Bayes factors** and **Group Bayes Factors**:

$$BF_{ij} = \frac{p(y|m_i)}{p(y|m_j)} \longrightarrow \log(BF_{ij}) = \log(p(y|m_i)) - \log(p(y|m_j))$$

$$GBF_{ij} = \prod_k BF_{ij}^{(k)} = \prod_k \frac{p(y_k|m_i)}{p(y_k|m_j)} \longrightarrow \log(GBF_{ij}) = \sum_k \log(BF_{ij}^{(k)})$$

Kass & Raftery, 1995, *J. Am. Stat. Assoc.*





# TRY TO SET UP A FIXED-EFFECTS BMS ANALYSIS

Implement the following two analyses:

- (1) Compute the individual log Bayes factors and plot the results. You should write your own MATLAB code for this. The negative free energies serve as our metric of model goodness and are stored in the "**DCM.F**" field.
- (2) Perform fixed-effects Bayesian model selection by computing the Group Bayes factor and/or the posterior model probabilities. You can either use the SPM GUI or write your own code. What do you notice when comparing the results from (1) and (2)?

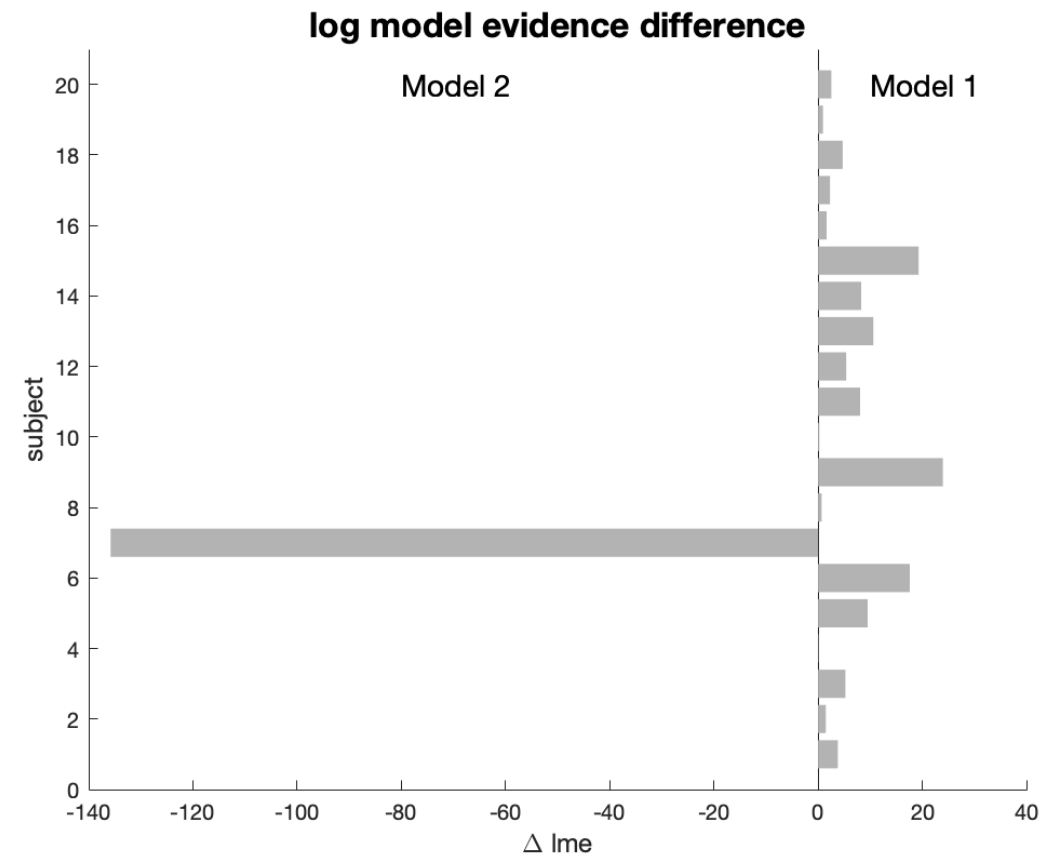
# TRY TO SET UP A FIXED-EFFECTS BMS ANALYSIS

(1) Implementation of individual log Group Bayes factor:

```
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3 % This exercise illustrates how to implement fixed-effects and random-
4 % effects Bayesian model selection (BMS), and how to perform Bayesian
5 % model averaging (BMA)
6 %
7
8 % results array for log model evidence
9 F = NaN(20,2);
10
11 % get the log model evidences (approximated by negative free energies)
12 for n = 1:20
13     if ( n < 10 )
14         n_text = ['0' num2str(n)];
15     else
16         n_text = num2str(n);
17     end
18     for m = 1:2
19         temp = load([pwd '/DCMs/DCM_model' num2str(m) '_' n_text '.mat']);
20         F(n,m) = temp.DCM.F;
21     end
22 end
23
24 % solution (a): compute subject-sepcific Bayes factors
25 -----
26
27 % compute Bayes factor
28 BF = exp(F(:,1)-F(:,2));
29
30
31 figure;
32 col = [0.7 0.7 0.7];
33 barh(F(:,1)-F(:,2), 'facecolor', col, 'edgecolor', 'none');
34 ylim([0 size(F,1)+1])
35 title('log model evidence difference', 'FontSize', 16)
36 ylabel('subject', 'FontSize', 12)
37 xlabel('\Delta lme', 'FontSize', 12)
38 text(-80,20, 'Model 2', 'FontSize', 14)
39 text(10,20, 'Model 1', 'FontSize', 14)
40 box off
41
```



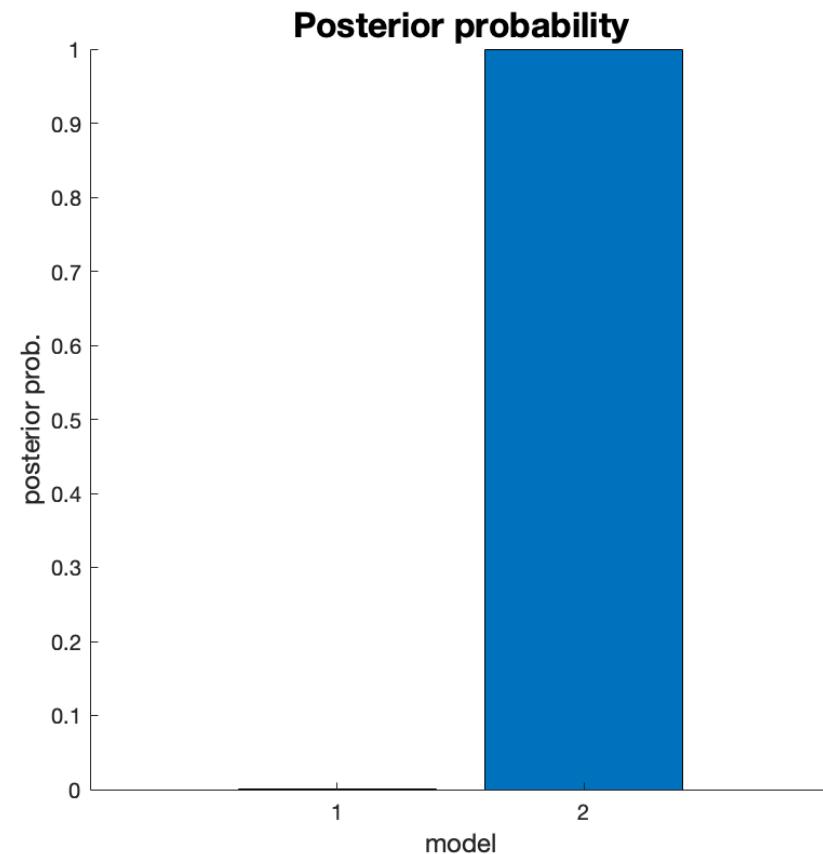
# TRY TO SET UP A FIXED-EFFECTS BMS ANALYSIS

(2) Implement fixed-effects Bayesian model selection:

```
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37 - xlabel('\Delta lme','FontSize',12)
38 - text(-80,20,'Model 2','FontSize',14)
39 - text(10,20,'Model 1','FontSize',14)
40 - box off
41
42
43 % solution (B): compute Group Bayes factor & posterior model probabilities
44 %-----
45
46 % compute the Group Bayes factor
47 sumF = sum(F,1);
48 GBF = exp(sumF - sumF(1));
49
50 % compute the posterior model probabilities
51 sumF = sumF - max(sumF);
52 pp = exp(sumF)./sum(exp(sumF));
53
54 % display posterior model probabilities
55 figure;
56 col = [0.6 0.6 0.6];
57 colormap(col);
58 bar(pp);
59 xlim([0 3])
60 set(gca,'xtick',[1 2])
61 title('Posterior probability','FontSize',16)
62 xlabel('model','FontSize',12)
63 ylabel('posterior prob.','FontSize',12)
64 axis square
65 box off
66
67
68 % solution (c): perform random effects BMS
69 %-----
70
71 % call spm_BMS
72 [alpha, exp_r, xp, pxp, bor] = spm_BMS(F, 1e6, 1, 0, 1, ones(1,size(F,2)));
73
74
75 % solution (d): compute BMA and evaluate posterior probability
```



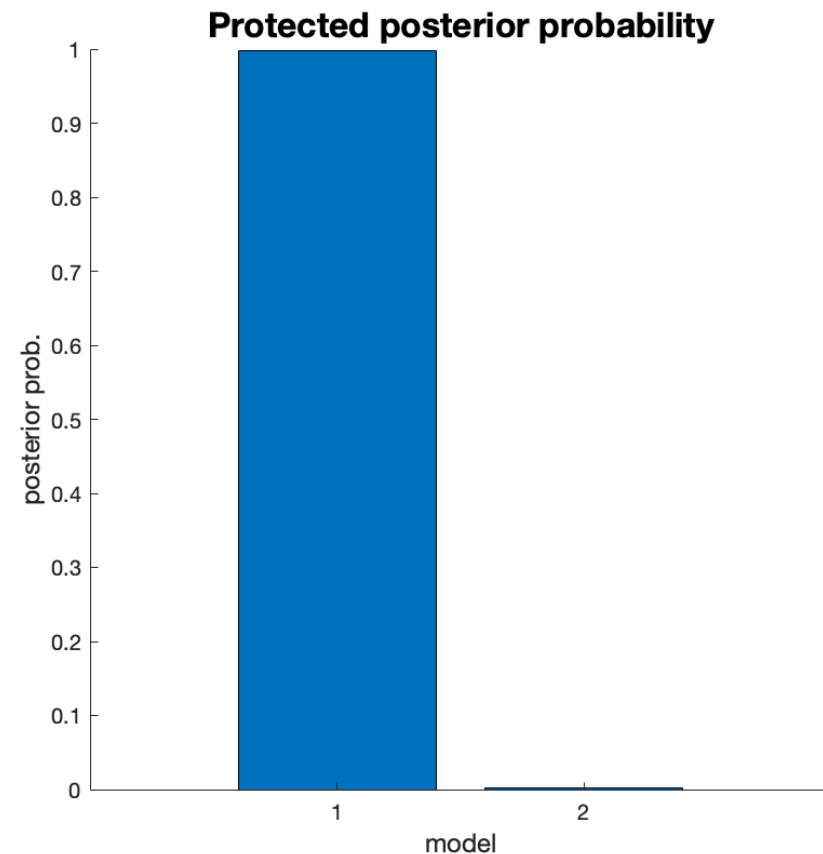
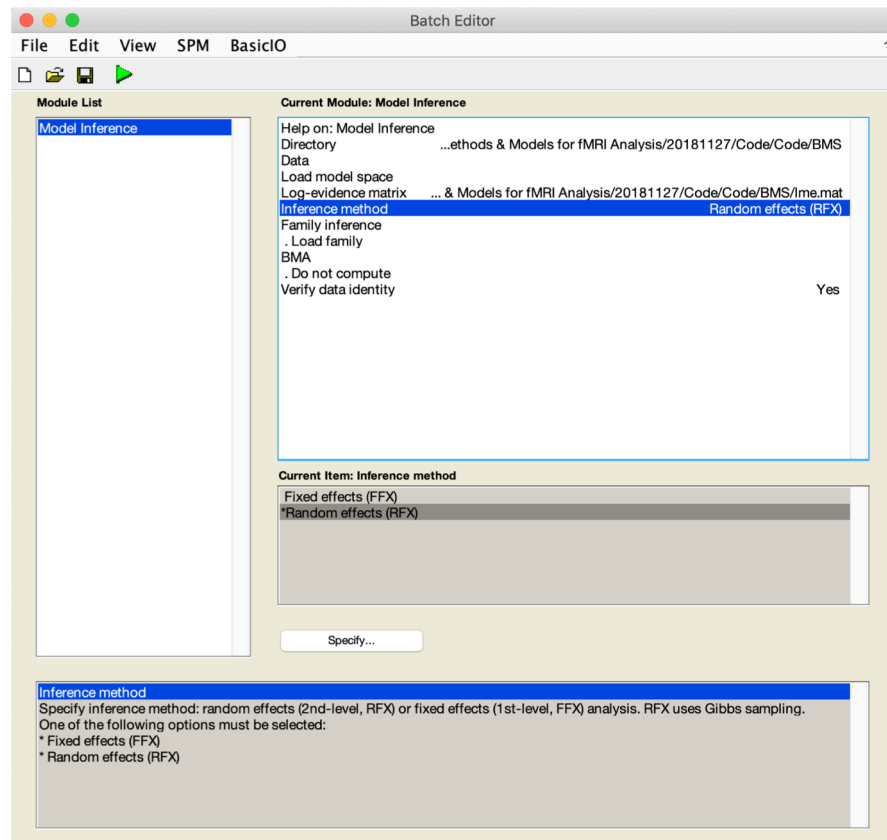
# TRY TO SET UP A RANDOM-EFFECTS BMS ANALYSIS

Implement the following analysis:

- (1) Perform random-effects Bayesian model selection by computing the protected exceedance probability. You can either use the SPM GUI or write your own code (*Hint: when writing your own code, use to function "spm\_BMS.m"*). What do you notice when comparing the results with the FFX-BMS results from before?

# TRY TO SET UP A RANDOM-EFFECTS BMS ANALYSIS

(1) Implement random-effects Bayesian model selection:



# TRY TO SET UP A RANDOM-EFFECTS BMA ANALYSIS

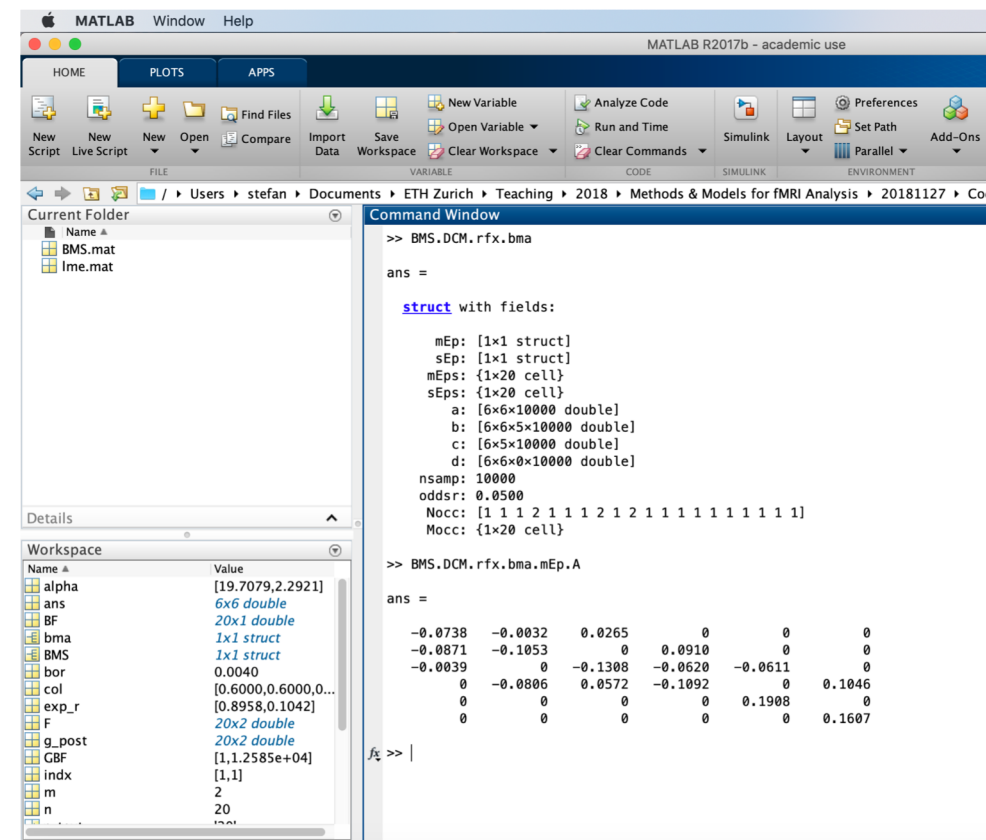
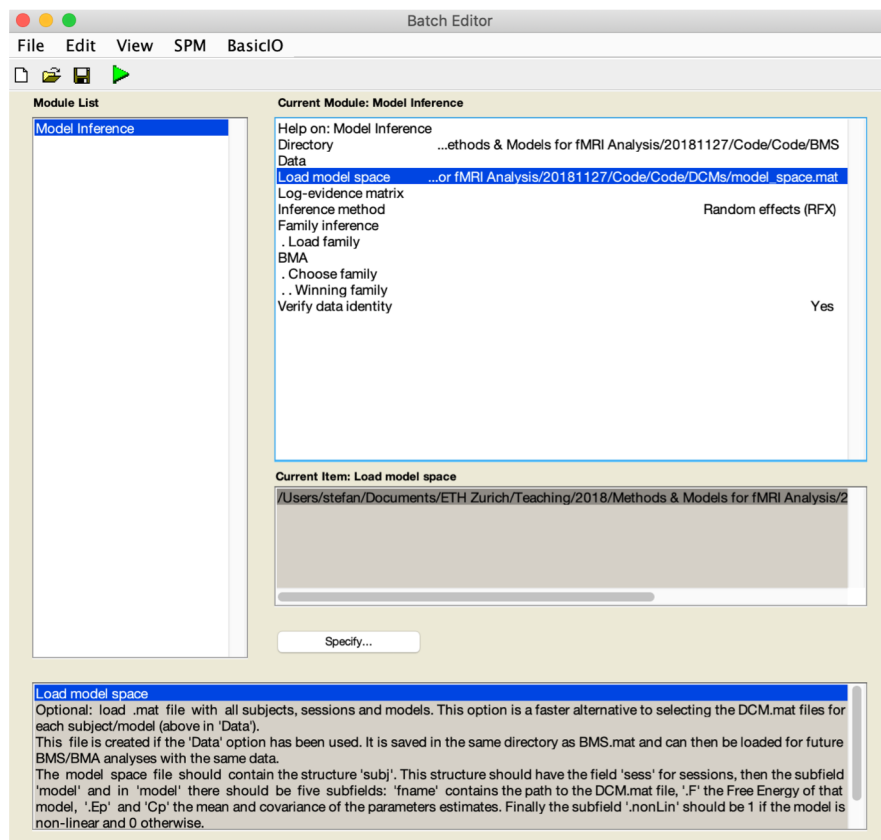
Implement the following analysis:

- (1) Perform random-effects Bayesian model averaging and compute the BMA posterior over model parameters. You can either use the SPM GUI or write your own code (*Hint: when writing your own code, use the functions "spm\_BMS\_gibbs.m" and "spm\_dcm\_bma.m"*).
- (2) Bonus: Test for which of the parameters, the connection strength of an "average" subject differs from zero. For this, use the group-level BMA results (*which are stored in  $mEp$  and  $sEp$* ) from step (1) to compute, for each connection, the posterior probability that the connection strength differs from zero (*Hint: the function "spm\_Ncdf.m" will be helpful for that purpose*).



# TRY TO SET UP A RANDOM-EFFECTS BMA ANALYSIS

(1) Implement random-effects Bayesian model averaging:



# TRY TO SET UP A RANDOM-EFFECTS BMA ANALYSIS

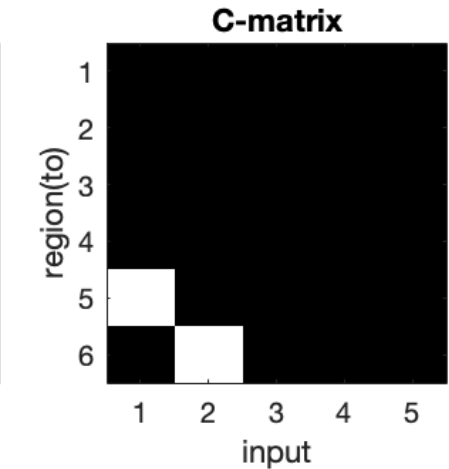
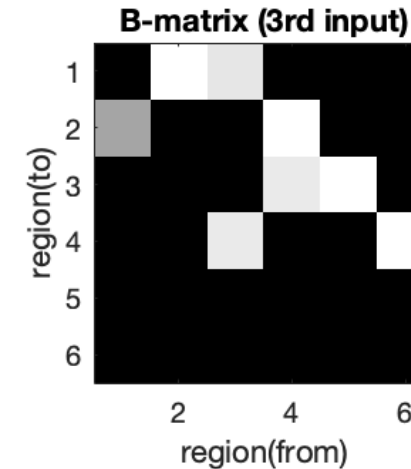
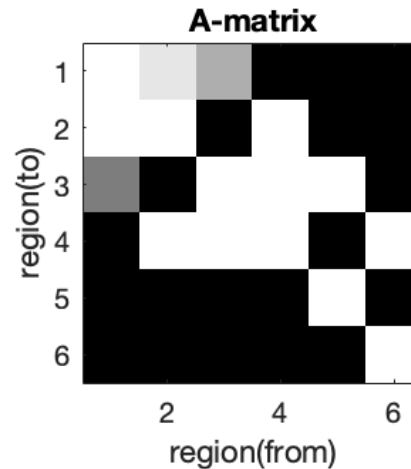
(2) Evaluate the posterior probability of each connection:

```
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125 bma = spm_dcm_bma(g_post,indx,subj,nsamp,odds_ratio);
126
127 % get posterior mean and covariance
128 Ep = bma.mEp;
129 Cp = spm_unvec((spm_vec(bma.sEp)).^2,bma.sEp);
130
131 % get prior means
132 model1 = load(pwd '/DCMs/DCM_model1_01.mat');
133 model2 = load(pwd '/DCMs/DCM_model2_01.mat');
134 DCM.a = model1.DCM.a | model2.DCM.a;
135 DCM.b = model1.DCM.b | model2.DCM.b;
136 DCM.c = model1.DCM.c | model2.DCM.c;
137 DCM.d = zeros(size(DCM.a,1),size(DCM.a,2),0);
138 [pE,~,~] = spm_dcm_fmri_priors(DCM.a,DCM.b,DCM.c,DCM.d);
139 T = full(spm_vec(pE));
140
141 % evaluate the posterior probabilities of model parameters
142 Pp = spm_unvec(1 - spm_Ncdf(T,abs(spm_vec(Ep)),spm_vec(Cp)),Ep);
143
144 % plot the posterior probabilities for A-matrix
145 figure,
146 subplot(1,3,1)
147 imagesc(Pp.A)
148 colormap('gray')
149 axis square
150 xlabel('region(from)')
151 ylabel('region(to)')
152 title('A-matrix')
153 caxis([0 1])
154
155 % plot the posterior probabilities for B-matrix (3rd input)
156 subplot(1,3,2)
157 imagesc(Pp.B(:, :, 3))
158 colormap('gray')
159 axis square
160 title('B-matrix (3rd input)')
161 xlabel('region(from)')
162 ylabel('region(to)')
163 caxis([0 1])
```





# QUESTIONS

REGARDING LECTURE OR PRACTICAL SESSION



THANK YOU FOR YOUR ATTENTION !

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